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Heat conduction with fractional Cattaneo–Christov upper-convective derivative flux model

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ABSTRACT

Fractional order derivatives are global operators for which the time fractional order derivative possesses memory character while the space ones reflects non-local behavior. In this paper, a new time and space fractional Cattaneo–Christov upper-convective derivative flux heat conduction model is suggested where the space fractional derivative is characterized by the weight coefficient of forward versus backward transition probability. Governing equation is formulated and solved by L1-approximation and shifted Grünwald formula. Results show that the fractional parameters, time and location parameters, relaxation parameter, weight coefficient and convection velocity have remarkable impacts on heat transfer characteristics. Temperature distribution profiles are monotonically decreasing in a concave form versus time fractional parameter with existing of relaxation parameter, while in a convex form with space fractional parameter evolution under three special conditions, i.e., the right region, the larger weight coefficient ($\gamma \geq 0.5$) and smaller convection parameter u.
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1. Introduction

A considerable attention has been devoted to heat conduction [1–3] due to its extensive application in widespread fields. The classical 1-D constitutive model to describe heat conduction is deduced by the Fourier's law [4] which provides a way to study heat conduction and becomes the basis to study the heat transfer process in the past few years. However, a paradox for the Fourier's model [5–7] is that it is felt instantly throughout the whole of the medium even for small times. This behavior contradicts the principle of causality [8,9] which issued an infinite propagation velocity. In order to overcome this problem, a modified constitutive model is proposed by Cattaneo [10] which takes the relaxation parameter into account.

The Cattaneo constitutive relation only involves partial time derivative, higher spatial gradients may be required [11] for a complete process. Revising the time derivative as the Oldroyds' upper-convected derivative, Christov [12] proposed the frame-indifferent generalization of Cattaneo model:

$$\mathbf{q} + \xi\left[\frac{\partial \mathbf{q}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{q} - \mathbf{q}\cdot\nabla\mathbf{V} + (\nabla\cdot\mathbf{V})\mathbf{q}\right] = -k \ \mathrm{grad} \ T, \tag{1}$$

where $\mathbf{q}$, $\mathbf{V}$, k, ξ and T refer to heat flux vector, velocity vector, thermal conductivity, relaxation parameter and temperature distribution function, respectively. The propagation velocity [13] is defined as $v = (D/\xi)^{1/2}$, and it reduces to the classical Fourier's law with an infinite propagation velocity for $\xi \to 0$. The new flux model satisfies the objectivity principle and attracts a large number of scholars' attention. Straughan [14] considered the thermal convection in a horizontal layer of incompressible Newtonian fluid with gravity acting downward. Using Cattaneo–Christov heat flux model, Han et al. [15] studied coupled flow and heat transfer of an upper-convected Maxwell fluid above a stretching plate, analyzing the dynamic property with different parameters effect and presenting a comparison of Fourier's Law and the Cattaneo–Christov heat flux model. Basing upon Cattaneo–Christov theory, Hayat et al. [16] considered temperature dependent thermal conductivity in stagnation point flow toward a nonlinear stretched surface with variable thickness, results showed that temperature profile decreases for higher thermal relaxation parameter. Sui et al. [17] introduced the Cattaneo–Christov model to study and analyze the boundary layer heat and mass transfer in upper-convected

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Maxwell nanofluid past a stretching sheet with slip velocity. More literature related to Cattaneo–Christov model can be seen in Refs. [18–21].

Fractional-order partial differential equation is a generalization and development of integer order one. The fractional derivative indicates that the position we consider is not only depended on its nearby positions but also on the whole positions, while the integer order operator is only a local one. For the time fractional derivative, Du et al. [22] indicated that its physical meaning is an index of memory. The space one reflects a non-local character and it can describe transfer process in a highly inhomogeneous medium more adequately by comparing with experiment data [23]. The study for the application of fractional derivative operator has attracted considerable attentions. Zaslavsky [24] reviewed the new concept of fractional kinetics for systems with Hamiltonian chaos, proving that fractional kinetics is valuable in different important physical phenomena. Henry et al. [25] introduced the temporal fractional cable equations to model electrotonic properties of spiny neuronal dendrites, predicting that postsynaptic potentials propagating can arrive at the soma faster along dendrites with larger spine densities and be sustained at higher levels over longer times. Chen et al. [26] proposed variable-order fractional derivative model which can agree significantly better with experimental data. For more references about the application of fractional operators, see in Refs. [27–29].

Motivated by above mentioned discussions, we firstly extend the study of heat conduction with time and space fractional Cattaneo-Christov equation. By considering the velocity as a constant and the generalized derivative of time [13] and space fractional order [30], Eq. (1) can be rewritten into the following one dimensional form:

$$q+\xi\left(\tau^{\alpha-1}\frac{\partial^\alpha q}{\partial t^\alpha}+u\frac{\partial q}{\partial x}\right)=-k\left(\gamma\frac{\partial^\beta T(x,t)}{\partial x^\beta}-(1-\gamma)\frac{\partial^\beta T(x,t)}{\partial(-x)^\beta}\right), \quad (2)$$

where τ is introduced to keep the dimension of constitutive equation balance and its dimension is "s", u is the convection velocity along the x direction, γ $(0\le\gamma\le 1)$ is the weight coefficient of forward versus backward transition probability, the symbol $\frac{\partial^\alpha}{\partial t^\alpha}$ is the Caputo's time fractional derivative [31] of order $\alpha(0<\alpha\le 1)$, defined as:

$$\frac{\partial^\alpha T(x,t)}{\partial t^\alpha}=\frac{1}{\Gamma(1-\alpha)}\int_0^t\frac{1}{(t-\tau)^\alpha}\frac{\partial T(x,\tau)}{\partial\tau}d\tau, \quad (3)$$

where the symbol $\Gamma(\cdot)$ represents the Euler gamma function.

The symbols $\frac{\partial^\beta}{\partial x^\beta}$ and $\frac{\partial^\beta}{\partial(-x)^\beta}$ are the left and right Riemann-Liouville fractional derivatives of order β $(0<\beta\le 1)$, the corresponding definitions [32] on a finite domain $[a,b]$ are given by:

$$\frac{\partial^\beta T(x,t)}{\partial x^\beta}=\frac{1}{\Gamma(1-\beta)}\frac{\partial}{\partial x}\int_a^x(x-\xi)^{-\beta}T(\xi,t)d\xi, \quad (4)$$

and

$$\frac{\partial^\beta T(x,t)}{\partial(-x)^\beta}=\frac{-1}{\Gamma(1-\beta)}\frac{\partial}{\partial x}\int_x^b(\xi-x)^{-\beta}T(\xi,t)d\xi, \quad (5)$$

respectively.

2. Mathematical formulation

First, we give the mass conservation equation:

$$c\rho\frac{\partial T}{\partial t}+c\rho u\frac{\partial T}{\partial x}+\mathrm{div}\ q=0, \quad (6)$$

where c and ρ are the specific heat capacity and mass density, respectively.

By the combination of (2) and (6), one arrives at the time and space fractional Cattaneo-Christov heat conduction equation:

$$\begin{aligned}&\xi\tau^{\alpha-1}\frac{\partial^{1+\alpha}T}{\partial t^{1+\alpha}}+\xi u\tau^{\alpha-1}\frac{\partial^{\alpha+1}T}{\partial x\partial t^\alpha}+\xi u\frac{\partial^2 T}{\partial x\partial t}+\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}\\&\quad+\xi u^2\frac{\partial^2 T}{\partial x^2}-D\frac{\partial}{\partial x}\left[\gamma\frac{\partial^\beta T}{\partial x^\beta}-(1-\gamma)\frac{\partial^\beta T}{\partial(-x)^\beta}\right]=0,\end{aligned} \quad (7)$$

with the initial and boundary conditions:

$$T(x,0)=\frac{1}{L^4}x^2(L-x)^2,\quad \frac{\partial T(x,0)}{\partial t}=0, \quad (8)$$

and

$$T(0,t)=T(L,t)=0, \quad (9)$$

respectively. Here $D=k/(c\rho)$ is the thermal diffusivity coefficient. For the sake of simplifying our study, the non-dimensional quantities are introduced:

$$t\to\tau t^*,\ x\to Lx^*,\xi\to\tau\xi^*,u\to\frac{L}{\tau}u^*,r\to\frac{L^{\beta+1}}{\tau D}r^*,1-r\to\frac{L^{\beta+1}}{\tau D}(1-r^*). \quad (10)$$

Submitting the non-dimensional quantities into (7)–(9), we can obtain the dimensionless governing equation with initial and boundary conditions (the superscript * is omitted for simplicity):

$$\begin{aligned}&\xi\frac{\partial^{1+\alpha}T}{\partial t^{1+\alpha}}+\xi u\frac{\partial^{\alpha+1}T}{\partial x\partial t^\alpha}+\xi u\frac{\partial^2 T}{\partial x\partial t}+\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+\xi u^2\frac{\partial^2 T}{\partial x^2}\\&\quad-\left[\gamma\frac{\partial^{\beta+1}T}{\partial x^{\beta+1}}+(1-\gamma)\frac{\partial^{\beta+1}T}{\partial(-x)^{\beta+1}}\right]=0,\end{aligned} \quad (11)$$

$$T(x,0)=x^2(1-x)^2,\quad \frac{\partial T(x,0)}{\partial t}=0, \quad (12)$$

$$T(0,t)=T(1,t)=0. \quad (13)$$

By setting $\beta=1$ and $u=0$, Eq. (11) reduces to the time fractional Cattaneo model [8] while Eq. (11) reduces to the classical heat conduction model [4] when $\beta=1$, $\xi=0$ and $u=0$.

3. Numerical discretization method

Firstly, we define $x_i=ih$ $(i=0,1,2,\ldots,m,\ mh=1)$ and $t_j=j\tau$ $(j=0,1,2,\ldots,n)$ where h is the grid size in space and τ is grid size in time. Prior to obtaining the numerical solution of Eq. (11), some useful definitions of difference scheme for the time and space fractional derivative are presented.

The Caputo fractional derivative of order $0<\alpha\le 1$ with respect to time at $t=t_j$ is approximated by L1-approximation [33], the discrete scheme is given as follows:

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