International Journal of Thermal Sciences xxx (2016) 1–11

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Validity of the macroscopic energy equation model for laminar flows through porous media: Developing and fully developed regions

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ARTICLE INFO

Article history:
Received 23 March 2016
Received in revised form
7 November 2016
Accepted 7 November 2016
Available online xxx

Keywords:
Porous media
Macroscopic energy equation
Cellular average
Volume average
Developing region
Fully developed region

ABSTRACT

The performance of the macroscopic energy equation model for laminar flows through porous media is tested and analyzed in this study. This is achieved by comparing the behavior of the model with data obtained from microscopic numerical simulations. These simulations correspond to a flow that is heated by a constant temperature boundary condition at the fluid-solid interface in a simple porous structure formed by staggered square cylinders. Specifically, laminar steady flow regimes with $Re_D = 1$, 10 and 75, Pe_D in the 10-10^4 range, and porosities between 55 and 95% are simulated. Applying the cellular average to the numerical solution allows obtaining the macroscopic temperature. Results clearly show the existence of two different regions at a macroscopic scale. At the entrance, there is a thermally developing region characterized by a rapid variation of the temperature with the streamwise coordinate. The second region is the fully developed region where the non-dimensional temperature varies exponentially with the streamwise coordinate. The length of the developing region is found to be relatively large for high Pe_D numbers allowing to conclude that the thermal entrance effect cannot be neglected in the use of macroscopic models for large Pe_D numbers. The model is also tested in the fully developed region showing excellent agreement with the data. It is found that the decay rate of the macroscopic temperature in this region scales with $Pe_D^{-0.8}$ and that the exponent is fairly independent of the porosity, flow conditions and fluid properties. Finally, it is shown that models that ignore the entrance region or neglect thermal dispersion are, in general, not valid.

1. Introduction

Porous structures, such as fibrous porous media, spatially periodic solid structures and open cell foams are commonly found in many engineering and industrial applications such as grain storage, transport in soils, flow diffusers, cryogenic tanks, core of nuclear reactors, radiators, and so on. In many of these applications and under laminar [1–3] or turbulent flow conditions [4–8], porous structures are placed to interchange heat efficiently. In this study, the focus is on the modeling of the heat transfer process in porous structures under laminar flow conditions [1–3]. Findings of the present study can be applied to the modeling of a variety of devices as for example compact heat exchangers, electronic components, cooling towers, packed bed reactors and power transformers. Additionally, results of this study can be applied to the modeling of heat transfer at micro and nanoscale (e.g. Ref. [9]).

The goal of porous media models is to satisfactorily represent the macroscopic scale of a physical process without representing explicitly the physics at the microscopic scale (i.e. the scale of the pore [10–12]). The volume-averaging technique, employed as a space averaging tool, has been successful in the derivation of porous media models from conservation laws at a microscopic scale [13]. In particular, the macroscopic energy equation (or a macroscopic transport equation for a passive scalar) was originally derived in Refs. [14,15] and variations of its original form are commonly found in the literature (e.g. Refs. [4,16,17]). There are two main coefficients regarding the heat transfer phenomenon at a macroscopic scale: the interfacial heat transfer coefficient and the thermal dispersion tensor [10]. During the last four decades, much effort was devoted to measure these parameters experimentally for different flow conditions, different fluids, and different geometries (e.g. Ref. [18]). Nowadays, extensive research is carried out to

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http://dx.doi.org/10.1016/j.ijthermalsci.2016.11.005
1290-0729/

Nomenclature

a_{sf}	interfacial area per unit volume
C_p	fluid specific heat
h_{sf}	interfacial or macroscopic heat transfer coefficient
k_f	fluid thermal conductivity
$k_{D\text{-}xx}$	dispersion coefficient in the streamwise direction
p	pore length scale
u	local streamwise velocity
x^*	streamwise non dimensional coordinate ($x/2H$)
D	square-edge length
H	REV's dimension (REV volume = $2H$ x H)
$L_{5\%}$	thermal entrance length
Nu_D	macroscopic Nusselt number ($h_{sf}\,D/k_f$)
Pr	Prandtl number
Pe	Péclet number
Pe_D	Péclet number based on the Darcy velocity and D
Re	Reynolds number
Re_D	Reynolds number based on the Darcy velocity and D
T_B	bulk temperature
T_i	inlet fluid temperature
T_w	wall temperature
$\overline{T}$	intrinsic (fluid) cellular average of temperature
TEL	thermal entrance length
$\overline{U}$	intrinsic (fluid) cellular average of streamwise velocity
U_D	Darcy velocity ($\overline{U}\phi$)
V	volume of the REV
V_f	fluid volume inside the REV

Greek symbols

α	decay rate of macroscopic temperature
α_2	decay rate of macroscopic temperature neglecting diffusion effects
ϕ	porosity
ν	kinematic fluid viscosity
θ_{m}	microscopic non dimensional temperature
θ	macroscopic (intrinsic cellular average) non dimensional temperature
ρ	fluid density
ξ	local coordinate

Additional notations

$\overline{\psi}^{VA}$	volume average of φ
$\overline{\psi}^{CA}$	cellular average of φ
$\overline{\psi}$	intrinsic (fluid) cellular average of φ
$^{i}\psi$	space fluctuation of φ
ΔL_{FD}	length difference between data and the fully developed model
$\%\Delta T_{FD}$	percentage temperature difference between data and the fully developed model

numerically compute these coefficients for different flow conditions, geometries, and fluid properties [19–21]. In general, these coefficients complete the macroscopic differential model for the transport of the macroscopic temperature.

As in the case of heat transfer in clear flows, the thermal entrance length (TEL) in a porous medium appears to be a relevant parameter to obtain. It is known, however, that the hydrodynamics entrance length in porous media, well discussed in Ref. [10], is of the size of a pore and therefore irrelevant at a macroscopic scale. Literature related to the TEL in porous media is scarce. Nevertheless, the experimental study of Wang and Du [22] provides some insight regarding this quantity. Measurements for the non-dimensional interfacial heat transfer coefficient (Nu_D) showed that this quantity evolves with the streamwise coordinate from a maximum at the entrance to a fully developed value in a distance equal to several equivalent diameters of the channel (the same general behavior is shown in Ref. [23]). In Ref. [24], Wang et al. analyzed further their data to find values for the TEL between 5 and 20 widths of the test section depending on the fluid properties and Reynolds number. Their results also showed that for the same flow conditions, the TEL is larger for the fluid with higher Prandt number. Wang et al. [24] also analyzed the data of [25] and [26] to calculate the TEL for two different experiments with air flowing through a porous medium. In this case, the TEL is between 2 and 4 widths of the test section. The numerical study of Imani et al. [27] also showed the dependence of the macroscopic heat transfer coefficient with the streamwise coordinate and the studies carried out by Teruel [23,29] and Teruel and Díaz [28] show the existence of a developing region that becomes larger when the Péclet number increases. Additionally, the experimental study of Han et al. [30] shows that the dispersivity in packed beds is a function of the streamwise position and that a developing length can be calculated measuring the spatial variation of this quantity. All the experimental evidence reviewed shows that the TEL is a macroscopic phenomenon that needs to be analyzed to find out its dependence on the characteristics of the porous medium, fluid properties and flow conditions.

Studies in the field also show that the use of the macroscopic energy equation is in general accompanied with some model assumptions. The most common assumption is to neglect the entrance effect in the thermal field. To neglect the streamwise thermal dispersion is also another findable assumption. For instance, in Refs. [31,32], analytical solutions of the macroscopic energy equation model are sought assuming constant values for the thermal dispersion and the interfacial heat transfer coefficients (i.e. the entrance effect is not considered). Another example of the use of such assumptions is the numerical study of Alfiere et al. [33]. The authors of this study are aware of the entrance effect but they acknowledged that to the best of their knowledge there is not an equation to calculate such effect in porous media flows. Regarding the assumption that neglects the streamwise thermal dispersion, this is done in Ref. [31] invoking a sufficiently high Péclet number or a highly convective flow. The study of Sano et al. [34] claimed an important statement on this regard: this coefficient, the thermal dispersion, may never be negligibly small for highly convective flows.

The main objective of this study is to test the validity of the macroscopic energy equation model in the laminar regime by comparing the solution of the macroscopic model with data obtained from microscopic numerical solutions in a porous structure. In particular, the study looks forward to give some insight over important aspects of the heat transfer process in laminar flows through porous media that, in the author's opinion, has been heretofore overlooked. One of them is the entrance effect in the thermal field. Another is the validity of the macroscopic energy equation model in the fully developed region. And finally, the assumption that the streamwise thermal dispersion can be neglected for high Péclet numbers flows. The work is organized as

Please cite this article in press as: F.E. Teruel, Validity of the macroscopic energy equation model for laminar flows through porous media: Developing and fully developed regions, International Journal of Thermal Sciences (2016), http://dx.doi.org/10.1016/j.ijthermalsci.2016.11.005