International Journal of Thermal Sciences xxx (2016) 1–10

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Heat conduction in two and three-phase media with solid spherical particles of the same diameter

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ARTICLE INFO

Article history:
Received 27 January 2016
Received in revised form
14 October 2016
Accepted 14 October 2016
Available online xxx

Keywords:
Two and three-phase media
Solid spherical particle with same diameter
Effective thermal conductivity
Thermal probe
Theoretical model
Parallel heat fluxes assumption

ABSTRACT

Heat conduction in two and three-phase media, composed of solid spherical particles of the same diameter, is investigated theoretically and experimentally. The theoretical model has no empirical constants and is based on the solution of the Fourier heat conduction equation, under the thermal assumption of parallel heat fluxes, in porous medium with porosity greater than 0.4764. The medium can be two-phase, fully saturated with water or air, or three-phase, partially saturated with water and air. The problem is investigated experimentally with the thermal probe, a vertical cylinder inserted into the glass beads of the diameter of 3 mm. The thermal probe, made in laboratory, has a diameter of 1.5 mm and length 150 mm, and contains an electric heater and a temperature sensor. The perfect line source theory in the transient regime is employed to measure the thermal conductivity of the water saturated glass beads. The theoretical results for the two-phase media, glass beads and water, are in good agreement with the experiments. The results of the theoretical model for the three-phase media, glass beads with water and air, are in good agreement with the experimental data of the literature, and are compared to several theoretical models of the literature.

1. Introduction

Heat transfer by conduction in multiple-phase media is an important problem in several different applications, e.g. agriculture, geology and geophysics, thermal insulation of buildings, geothermal engineering, filtration processes, ground water pollution and heat storage systems, cooling of nuclear reactors, catalytic reactors, heat exchangers, high-voltage underground cables, radioactive waste storage facilities below the ground.

The solution of this kind of problem has been carried out employing several approaches which are described in the following. Since the heat conduction equation has been discovered first, the first kind of approach has been the direct solution of the basic heat transfer conduction equation, the Fourier law. For an idealized two phase media composed of a cubic solid and a fluid around, the simplest solution of the Fourier equation is to assume a one-dimensional heat transfer through the unit cell, under the hypothesis of temperature or heat flux distribution [1]. The effective thermal conductivities of the two phase media, calculated according to the two thermal assumptions, are the upper and lower bounds on the effective thermal conductivity of all normally distributed mixtures, [1,2]. The difficulty in the solution of the Fourier equation arises when the knowledge of the shape, size, location and thermal properties of each particle is uncertain, or when shape and location are very complicated. For this reason the use of the electro-magnetic equations, specifically the Ohm law, has received a great attention, and the approach of the so-called electrical analogy is resumed by the Maxwell approach [3], for sufficiently dilute dispersion of the spherical particles, later on extended to ellipsoidal particles and generalized to the case of n dispersed phases embedded in one continuous phase, [4]. Also the electrical analogy presents some limitations and the third approach to be mentioned is the empirical one. A limited number of papers is possible to mention here in order to review briefly the attention that each of the three approaches has received.

Starting the review from the electrical analogy, the extension of the Maxwell approach [3], sees the empirical proposal of an expression for the electrical permittivity of a composite, consisting of any number of randomly mixed components [5], the application to the physics of plant environment [6], and to three-phase mixture in spherical coordinates, [7]. The Maxwell approach, valid for a solid fraction tending to zero, or for porosity tending to 1, was extended

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http://dx.doi.org/10.1016/j.ijthermalsci.2016.10.022
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Please cite this article in press as: S. Corasaniti, F. Gori, Heat conduction in two and three-phase media with solid spherical particles of the same diameter, International Journal of Thermal Sciences (2016), http://dx.doi.org/10.1016/j.ijthermalsci.2016.10.022

by Lord Rayleigh to smaller porosities [8], later on corrected numerically [9], and applied to the case of a cubic array of uniform size spheres [10], with the proposal of a different potential function and a different final equation. The thermal resistance approach has been employed in Ref. [11] to calculate the effective thermal conductivity of porous materials taking into account the effects of temperature, porosity, material skeleton thermal conductivity, gas in the pore, pore size and other kinds of heat transfer mechanisms.

The empirical approach can be resumed by the weighted mean. Since the series distribution of the two phase in the medium corresponds to the weighted harmonic mean and the parallel distribution to the weighted arithmetic mean, the approach of the weighted geometric mean, for the effective medium approximation, can be considered empirical because it does not make reference to a physical principle, but simply gives results comprised between the other two means. The weighted geometric mean approach has been used in Ref. [12] to compare their experimental results. The empirical model of Lichtenecker [5] has been applied to the effective thermal conductivity of three-component composites, expressed by the volumetric content of each one, [13]. The macroscopic properties of randomly inhomogeneous materials, having specific geometry of the constituents, was investigated in Ref. [14] to take into account the different shapes of the inclusions embedded in the matrix, while the effective media formation and conduction through unsaturated granular materials was studied in Ref. [15]. Models of weighted effective medium approximations were developed to estimate the thermal conductivity of random composites with different materials and shapes [16]. Inter-particle contact heat transfer in soil systems at moderate temperatures was studied in Ref. [17]. An empirical model for the investigation of a packed bed of spherical particles, with points of contacts in the heat flow direction was employed in Ref. [18].

Many papers of the literature have solved the heat conduction equation in multiple phase systems taking into account also other phenomena, as thermal radiation. Kunii and Smith [19] predicted the effective thermal conductivity of beds of unconsolidated particles containing stagnant fluid and neglecting radiation. The effect of air pressure on the effective thermal conductivity of a bed of spherical particles was investigated in Ref. [20], while the thermal conductivity of granular materials was studied in Ref. [21]. A model for the evaluation of the effective thermal conductivity of unsaturated frozen soils was proposed in Ref. [22]. Two-phase media with the solid particles of the same diameter and porosities smaller than 0.4764 were modelled in Ref. [23]. The theoretical approach of [22] was later on extended to bricks [24]. Heat transfer in a packed bed with wall effect was studied in Ref. [25], showing that the heat conduction characteristics are function of the ratio of the particle diameter to the characteristic length of the geometry and to the ratio of the thermal conductivity of the fluid to that of the solid phase. The approach of [25] was modified for stagnant thermal conductivity of porous media [26,27], where a unit cell model was used to determine the effective thermal conductivity of bi-dispersed porous media, based on the lumped parameter method. The theoretical prediction of the thermal conductivity of soils at moderately high temperatures was done in Ref. [28]. A model to evaluate the radiation conductivity tensor was developed in Ref. [29] for porous media composed by spheres or cylinders, and the numerical results showed that the radiation contribution can influence the temperature distribution across the particle surface. Theoretical predictions of the thermal conductivity of two-and three-phase water/olivine systems were compared to experiments in Ref. [30]. The approach of [22] was extended to the investigation of extraterrestrial soil analogues, as in planets and comets [31–34], frozen meats [35], soils at elevated temperatures [36]. Felske [37] considered the spheres as homogeneous and the spherical unit composed of a composite particle surrounded by a layer of the continuous medium with a contact resistance between particle and the continuum negligible, and an exact series solution of the heat conduction equation was obtained for the temperature distribution in each phase. The effect of solid thermal conductivity and particle-particle contact on thermo-diffusion processes was investigated in Ref. [38], showing that for non-consolidates porous material, composed of spherical particles, the thermal conductivity ratio does not influence the thermal diffusion process. The approach of [22] was applied to three-phase media in Ref. [39], to composite materials [40–42], and to three-phase soils [43]. Minimization of thermal resistance in an air cooled porous matrix, made of solid spheres with heat generation, was investigated in Ref. [44]. A theoretical model was developed in Ref. [45] to take into account the decrease of the lattice thermal conductivity in a porous medium with inhomogeneous porosity. Porous media with various pore groups and different diameters of pores leads to the significant decrease in the lattice thermal conductivity, compared, not only to bulk materials with zero porosity but also to materials with homogeneous porosity. The approach of [22] has been extended recently to composites [46,47].

The aim of the present paper is to present a new theoretical model to evaluate the effective thermal conductivity of two and three-phase media, made of solid spherical particles of the same diameter, and with porosity greater than 0.4764, without any empirical constant, by extending the previous approach [23]. The paper compares the theoretical results with some experimental measurements carried on by the authors for two-phase media.

2. Theoretical model

2.1. Two-phase media with solid spherical particles

The unit cell of the medium, for porosities greater than 0.4764, has a rhombic base and contains a solid spherical particle of radius R_1 in the center. Fig. 1 presents the unit cell in three-dimensions. The angle β of Fig. 1a is variable with the porosity and, for $\beta = 90°$, the porosity of the cell is 0.4764 according to its definition:

$$\varepsilon = \frac{V_{cell} - V_{solid}}{V_{cell}} = 1 - \frac{\pi}{6} = 0.4764 \tag{1}$$

where $V_{solid} = \frac{4}{3}\pi R_1^3$, and $V_{cell} = 8R_1^3$. If $\beta < 90°$ the volume of the cell becomes:

$$V_{cell} = 2R_1 \cdot 2R_1 \cdot 2R_1 / \sin\beta = 8R_1^3 \Big/ \sin\beta, \tag{2}$$

where the particle volume is $V_{solid} = \frac{4}{3}\pi R_1^3$ and the porosity is equal to the ratio between the volume of the voids and the total volume of the cell

$$\varepsilon = \frac{V_{cell} - V_{solid}}{V_{cell}} = 1 - \frac{\pi}{6}\sin\beta > 0.4764 \tag{3}$$

The rhombic base of the cell of Fig. 1a has the side equal to a

$$a = \frac{2R_1}{\sin(\beta)} \tag{4}$$

The area of the rhombic base, S, is

$$S = a^2 \sin(\beta) = (2R_1)^2 \Big/ \sin(\beta) \tag{5}$$

while the total volume of the cell is

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