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The effect of yield stress on pipe flow turbulence for generalised newtonian fluids

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ABSTRACT

The effect of modifying yield stress on turbulent pipe flow of generalised Newtonian fluids at a friction Reynolds number of 323 is investigated using direct numerical simulations. Simulations are carried out for Bingham and Herschel–Bulkley fluids with the yield stress varying from 0% to 20% of the mean wall shear stress. Results show that the effect of increasing yield stress is mostly similar to shear thinning in power-law fluids. The turbulent viscous stress which arises due to viscosity fluctuations is negative for a yield stress fluid and is higher in magnitude for higher yield stress. An analysis of the turbulent kinetic energy budget showed that the effect of yield stress is mainly significant near the wall for $y^+ \lesssim 60$ which was also seen for shear-thinning power-law fluids at similar Re_τ. Additional shear thinning enhances the yield stress effect. The main difference between shear thinning and yield stress is that the effect of yield stress is maximum outside the viscous sublayer whereas shear thinning has a more significant effect inside the viscous sublayer.

1. Introduction

Many fluids found in industry and in nature do not show a uniform viscosity. These fluids are called non-Newtonian fluids. Generalised Newtonian fluids is a class of non-Newtonian fluids for which the rheology can be modelled via the generalised Newtonian (GN) assumption

$$\boldsymbol{\tau} = \rho\nu(\dot{\gamma})\boldsymbol{s}. \tag{1}$$

Here $\boldsymbol{\tau}$ is the shear stress tensor, ρ is fluid density, ν is fluid kinematic viscosity (also called the effective viscosity), shear rate $\dot{\gamma} = (2\boldsymbol{s}{:}\,\boldsymbol{s})^{1/2}$ is the second invariant of the strain rate tensor $\boldsymbol{s} = [(\nabla\boldsymbol{v}) + (\nabla\boldsymbol{v})]/2$ and $\boldsymbol{v}$ is the velocity. The GN assumption also implies an isotropic, time-independent viscosity and an instantaneous response of the fluid to the applied shear stress. Many GN fluids show yield stress i.e. they do not flow until the shear stress exceeds a minimum value (yield stress). Mining slurries, particle suspensions, waste water sludge, toothpaste, cements, tomato ketchup, melted chocolate are examples of yield stress GN fluids. Recently the Journal of non-Newtonian fluid mechanics published a special issue (the first special virtual issue, 2014) focusing only on yield stress fluids, which shows the continuing research interest in these fluids.

The effective viscosity of a GN fluid is defined via a rheology model. There are various rheology models available for yield stress GN fluids [1,2] in which the Herschel–Bulkley rheology model is a widely used model which relates the fluid kinematic viscosity to the shear rate via

$$\nu = \rho^{-1}(\tau_y/\dot{\gamma} + K\dot{\gamma}^{n-1}). \tag{2}$$

Here, the yield stress τ_y, fluid consistency K and the flow index n are model parameters. For $n < 1$, Eq. (2) represents shear-thinning behaviour i.e. the fluid viscosity decreases with increasing shear rate. With $n = 1.0$, Eq. (2) reduces to the Bingham rheology model $\nu = \rho^{-1}(\tau_y/\dot{\gamma} + K)$, with K known as plastic viscosity. When $\tau_y = 0$, Eq. (2) reduces to a power-law rheology model which represents purely shear-thinning (or thickening) behaviour. It is worth noting that rheology model parameters are usually determined via regression using an experimentally measured shear rheogram (one dimensional shear stress versus shear rate measurements) and have no intrinsic physical meaning. In the following Herschel–Bulkley (HB), Bingham and power-law (PL) fluids are those whose rheology can be well-modelled with the corresponding model.

Turbulent pipe flow is an important class of wall bounded turbulent flows. A pipe flow has the characteristic feature of an enclosed geometry, making it easiest to realise in experiments compared to other wall bounded flows such as channel and boundary layer flows [3]. It also has a direct and familiar application – pipeline transport which is very common at mining and waster water treatment sites to transport slurries which can show both yield stress and shear-thinning behaviour and the rheology of which can be modelled via the HB rheology model [4]. In spite of wide applications, studies of turbulent pipe flow of HB

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fluids are limited [5–9].

The HB rheology combines the effect of yield stress and shear thinning (or thickening). It has been shown that the HB rheology delays transition to turbulence to a higher Reynolds number and reduces the turbulent friction factor $f = 2\tau_w/\rho U_b^2$ under fully developed turbulent conditions [5,7,8]. The HB rheology increases the turbulent anisotropy by increasing the velocity fluctuations in the axial direction but decreasing the same in the radial and the azimuthal direction compared to Newtonian fluid. These trends are consistent with those of a shear-thinning fluid alone [7], therefore, the effect of the yield stress alone on turbulent pipe flow dynamics is not clear. Peinxinho et al. [10] claimed that the yield stress did not have a significant effect in the turbulent regime, however, fluids used in that study showed some viscoelastic behaviour, and the effect of yield stress is less clear.

In real fluids the rheology model parameters are generally coupled with each other. The rheology arised from surface effects in fine particle suspensions and polymer interaction in polymer based lab fluids. The rheology is changed by modifying concentration and potentially pH, however, such changes generally modify all rheology parameters [11–13], and it is difficult to change just one while keeping others constant. This makes experimental investigation of individually varying rheology model parameters impossible. Numerical simulations, especially direct numerical simulations (DNS) are promising in this aspect and have been used in the past in turbulent flow studies of HB fluids [7,8]. Although significant discrepancies had been observed between numerical and experimental results [8], recently these have been shown to be caused by a lack of high shear rate data used in rheology characterisation [14]. DNS provides a detailed picture of the flow and once validated, can be reliably used to understand the effect of individually varying rheology model parameters. DNS has other advantage that unlike other numerical techniques such as Reynolds averaged Navier–Stokes (RANS) and large eddy simulations (LES), it does not require any special model to capture the flow at small length scales. There have been some efforts in developing RANS and LES models for GN fluids [9,15–17] but there are no universally accepted models yet available.

Turbulent flows present a wide range of length scales (eddy sizes) and the HB rheology decreases the range of the length scales in the flow [7]. Earlier DNS studies of HB fluids [7,8] considered flow indices $n = 0.52$ and 0.6 with a maximum Reynolds number Re_G (defined in Section 2.2) of 8000. However, the flows showed some transitional behaviour especially for $n = 0.52$. To overcome this limitation, the current study considers a slightly higher Reynolds number $Re_G \approx 11000$ ($Re_\tau = 323$) to study the effect of yield stress τ_y on a turbulent pipe flow. To study the effect of varying τ_y alone, simulations are run using the Bingham rheology model with the yield stress varying from 0% to 20% of the mean wall shear stress. Additional simulations with the HB rheology model are run to study the effect of additional shear thinning. The results of mean flow, turbulence intensities and the turbulent kinetic energy budget are analysed and compared with those of Newtonian and PL fluids. The key findings are that the effect of yield stress is confined to the near wall region and unlike shear thinning it affects flow most noticeably outside the viscous sublayer.

2. Methodology

2.1. Numerical method

The numerical method used here is identical to that used in Rudman & Blackburn [7,8,18]. Here we briefly review the simulation methodology. A nodal spectral element-Fourier DNS code is used to solve the governing equations (Eq. 3) for an incompressible fluid with a spatially varying viscosity.

$$D\boldsymbol{v}/Dt = \rho^{-1}(-\nabla p + \nabla\cdot\boldsymbol{\tau} + \rho\boldsymbol{g}), \quad \text{with} \quad \nabla\cdot\boldsymbol{v} = 0 \tag{3}$$

where $\boldsymbol{v}$ is the velocity vector, p is the static pressure, $\boldsymbol{\tau}$ is the stress tensor and $\rho\boldsymbol{g}$ is the body force. For ease of notation, we divide p, $\boldsymbol{\tau}$ and $\rho\boldsymbol{g}$ in Eq. (3) by the constant fluid density ρ, but refer to them as pressure, stress and body force respectively. The body force $\boldsymbol{g}$ is set equal to the mean axial pressure gradient. The modified shear stress tensor, $\boldsymbol{\tau}/\rho$, is modelled via the GN assumption (Eq. 1) and the fluid viscosity, $\nu(\dot{\gamma})$, is modelled via the HB rheology model. The governing equations are solved in Cartesian coordinates where the pipe cross section (x-y plane) is discretized using spectral elements as shown in Fig. 1, while Fourier expansion is used in the axial (z) direction. Results are later transformed for presentation in cylindrical coordinates. The code has been validated for DNS of pipe flow of turbulent Newtonian fluids [18] and non-Newtonian fluids [8,14]. For more details of the simulations code we refer the reader to [7,8,19].

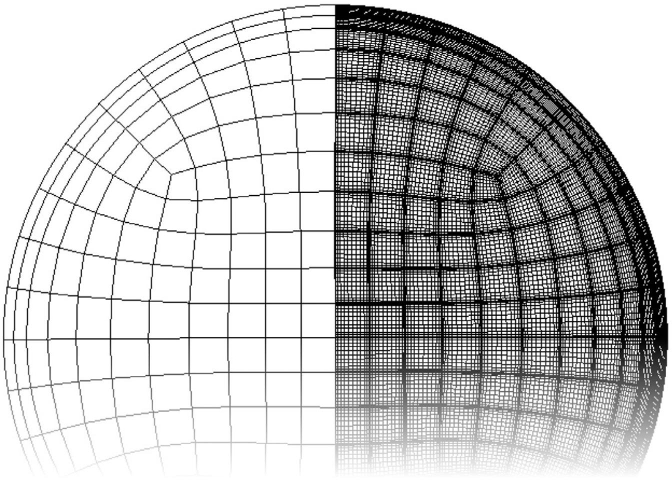

Fig. 1. Detail of a spectral-element mesh used to discretise pipe cross-section, illustrating grid nodes for 12th-order element interpolation functions, $N_p = 12$.

2.2. Reynolds number

The non-uniform viscosity of GN fluids makes the choice of an appropriate viscosity scale unclear. We choose the nominal wall viscosity, ν_w, for the viscosity scale as discussed by Rudman et al. [7]. For the HB rheology model, ν_w is given as:

$$\nu_w = \frac{1}{\rho}\frac{K^{1/n}\tau_w}{(\tau_w - \tau_y)^{1/n}}. \tag{4}$$

Here τ_w is the mean wall shear stress which is determined from the mean axial pressure gradient $\partial P/\partial z$ as:

$$\tau_w = (R/2)(\partial P/\partial z) \tag{5}$$

where R is the pipe radius. Using ν_w, pipe diameter $D = 2R$, bulk flow velocity U_b (flow rate per unit area) and the friction velocity $u^* = (\tau_w/\rho)^{1/2}$, we define the generalised Reynolds number Re_G and the friction Reynolds number Re_τ as:

$$Re_G = U_b D/\nu_w \quad \text{and} \quad Re_\tau = u^* R/\nu_w. \tag{6}$$

2.3. Simulation parameters and non-dimensional variables

Simulations are run for a fixed friction Reynolds number Re_τ of 323 which is equivalent to a generalised Reynolds number $Re_G \approx 11,000$ (Re_G slightly varies with n and τ_y, see Table 1). The effect of τ_y alone is studied with $n = 1.0$ (Bingham rheology model) and τ_y varying from 0% (Newtonian) to 20% of τ_w. Additional simulations with $n = 0.8$ and $\tau_y = 0\%$ (PL) and 10% of τ_w are carried out to study the additional effect of shear thinning.

Results are normalised using the friction velocity $u^* = (\tau_w/\rho)^{1/2}$ for the velocity scale and ν_w/u^* for the length scale. Hence the distance