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Stability of shear banded flow for a viscoelastic constitutive model with thixotropic yield stress behavior

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ABSTRACT

A viscoelastic constitutive model which combines the partially extending strand convection model and a Newtonian solvent is used in the regime of large relaxation time. Prior work on one dimensional time-dependent solutions at prescribed shear stress predicts some of the features expected of thixotropic yield stress fluids, such as delayed yielding. In this paper, we present the linear stability analysis of two-dimensional plane Couette flow, for parameter regimes that support a two-layer arrangement consisting of an unyielded layer and a yielded layer. Asymptotic analysis and computational techniques are applied. We find that the one layer yielded flow can have bulk instabilities which also emerge in the two-layer flow. Bulk instabilities in the yielded phase appear not to have been observed in prior literature. For some parameters, an interfacial mode is unstable and is driven by the normal stress difference across the interface. The yielded zone has the higher first normal stress difference, as for the well-studied Johnson–Segalman model. In order to assess the importance of the sign of the first normal stress difference at the interface, we specifically design a modification to the model to reverse the sign. It is found that instabilities still occur.

1. Introduction

Shear banding is the coexistence of high shear rate (“yielded”) and low shear rate (“unyielded”) zones at the same shear stress, and occurs in many complex fluids such as wormlike micellar solutions, pastes and suspensions [1–6]. In this work, we are interested in instabilities of shear banded flows. We find that these arise both from the bulk and at the interface between the two zones.

In the limit of a long relaxation time, the PECN model (partially extending strand convection model and Newtonian solvent) predicts certain characteristic behaviors of thixotropic yield stress fluids [7], allowing a precise mathematical expression of the idea that yield stress behavior is really a limit of extremely long relaxation time [8]. The yielding and unyielding hehavior of the model under an imposed constant stress has been analyzed in prior work [7], and we briefly summarize the results. Specifically, the model equations contain a small paramter ϵ, which physically corresponds to a ratio of retardation time to relaxation time; in our nondimensionalization below, time is scaled with the retardation time. If we formally set $\epsilon = 0$, the PECN model reduces to a nonlinear elastic model, and this phase describes the initial evolution from equilibrium when a stress is suddenly imposed. The elastic shear stress assumes a maximum at a certain value of shear deformation and then decreases to zero as the shear is increased further.

With small but non-zero ϵ, the steady shear response is non-monotone, with a stress maximum of order 1 at a shear rate of order ϵ, and a stress minimum of order $\epsilon^{1/4}$ at a shear rate of order $\epsilon^{1/4}$. For nonzero ϵ, there is no “true” yield stress; the shear rate in the “unyielded” regime is nonzero but small, of order ϵ. Thus, simply setting $\epsilon = 0$ in the governing equations gives only an incomplete picture of the dynamics, because different asymptotic regimes that depend on ϵ arise when long times and/or large deformations arise. These different regimes are considered in detail in [7]. One of the results of the interplay of these dynamic regimes is delayed yielding: The value of the stress maximum in steady shear flow is lower (by a factor $\sqrt{2}$) than the elastic shear stress maximum. If the imposed shear stress is higher than the elastic maximum, yielding will happen immediately, but if the imposed stress lies between the steady and elastic maxima, yielding will happen eventually, on a long time scale of order $1/\epsilon$. In summary, the small ϵ asymptotic analysis shows that flow is induced if either the shear stress exceeds the elastic stress maximum (immediate yielding), or the maximum on the steady flow curve (delayed yielding). Once flow begins, the yield stress is lowered, i.e. there is yield stress hysteresis.

If the shear stress is suddenly removed in an established yielded flow, the motion will cease quickly, but the viscosity will

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return to its equilibrium value (which is of order $1/\epsilon$) only over a timescale of order $1/\epsilon$, i.e. we observe thixotropic behavior. This is documented in [9].

In a shear rate controlled experiment, there is no stable homogeneous steady flow if the imposed shear rate falls into the interval where the steady response curve is decreasing. However, there are shear banded solutions, where two or more layers are formed, and in each layer the shear rate is on an increasing part of the steady flow curve. The shear stress is continuous across the boundary between layers, but the normal stress is not.

Parallel flows consisting of layers of two different viscoelastic fluids may have interfacial instabilities driven by a normal stress jump at the interface. Such instabilities were first found in the flow of two upper convected Maxwell fluids by Renardy [10] and Chen [11]. A shear banded flow, resulting from a non-monotone constitutive curve is analogous to this; although the two layers are the same fluid, they have different shear rates and normal stresses, and the interface can be treated as a material surface. Renardy [12] investigated the shear-banded flow of a Johnson–Segalman fluid and found interfacial instabilities driven by a normal stress jump. Her analysis was later expanded, most notably by a study of the long wave limit and by the inclusion of stress diffusion at the interface [13–15]. Even earlier, McLeish [16] found a normal-stress driven interfacial instability in shear-banded flow of a Doi-Edwards fluid. Rather than a full linear stabiity analysis, he used a simplified set of equations which was intended to capture the essential physics of long-wave instabilities.

There is much work based on the Johnson–Segalman model [13–15,17–19]. On the other hand, there are other established models that display non-monotone constitutive behavior and shear banded flows; for example, PECN is a Newtonian solvent model combined with a "partially extending strand convection" model, originally introduced by Larson [20] for entangled polymer melts. The Vasquez-Cook-McKinley (VCM) model is a refinement of this, which has had success in modeling the behavior of wormlike micellar solutions [21].

Fluids which show shear banding include wormlike micelles, some polymers and soft glassy materials [3]. Instabilities in shear banded flows have been observed in wormlike micelles. Experiments are typically done in curved geometries, where, aside from the interfacial instability there is also the possibility of a viscoelastic Taylor instability. It is believed that the Taylor instability is the dominant mechanism in most of the experimental observations [3]. There are, however, some experiments showing instabilities which have been attributed to interfacial mechanisms; we cite in particular [22] and [23]. Both first and second normal stress jumps can cause interfacial instabilities. The instability observed in [22] leads to interface oscillations in the spanwise direction, as would be expected from a second normal stress effect. In [22], the authors link the experimental observations to an analysis of the Johnson–Segalman model [18], which shows that although streamwise waves have a larger linear growth rate, nonlinear interactions ultimately favor spanwise perturbations. However, Fig. 2 in [22] shows no evidence of transient growth of any streamwise waves. The experiments of [23] show streamwise corrugations of the interface as would result from an instability driven by a jump in the first normal stress difference. We note that, unlike the Johnson–Segalman model, neither the PEC nor the VCM model has a second normal stress difference.

The analysis presented below shows that shear banded flows of the PECN model have instabilities. We are not aware of prior studies of the PECN model per se. However, [24] studied the stability of diffusive interfaces for the VCM model, and interfacial instabilities were found.

Our analysis shows that actually there are instabilities in the yielded phase even when there is no interface. Where they are

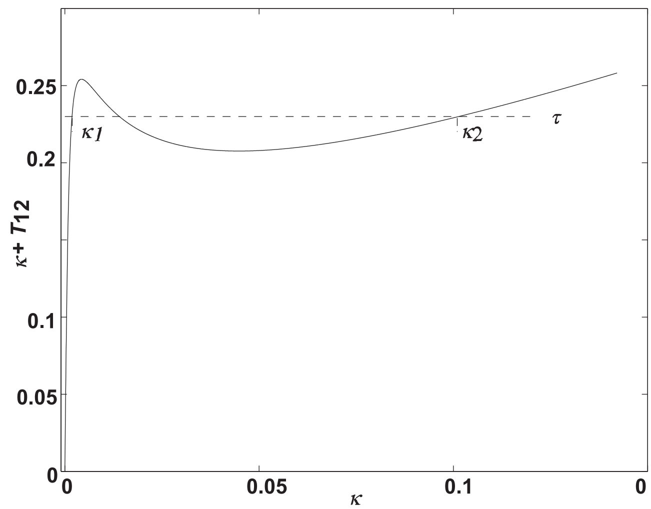

Fig. 1. Total shear stress $\kappa(s)+T_{12}(s)$ as a function of shear rate $\kappa(s)$. The dashed line denotes the prescribed total shear stress τ, and intersects the increasing parts of the curve at κ_1 and κ_2. The specific parameters for the PECN model are $\alpha=-1$, $\epsilon=0.001$.

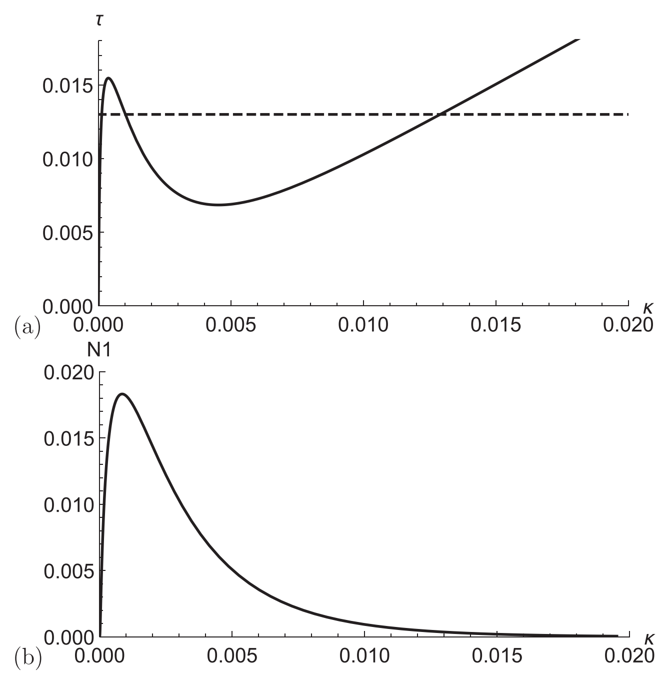

Fig. 2. Altered PEC model with $\psi(s)=\exp(-s)$, $\alpha=-2.8$, $\epsilon=0.001$. (a) Steady state solutions for total shear stress τ versus shear rate κ $(=U'(y))$. (b) Steady state solutions for the first normal stress difference N_1 versus κ.

present, these bulk instabilities, rather than interfacial instabilities, turn out to be the dominant mechanism of instability in shear-banded flows. The modes responsible for these bulk instabilities bifurcate from the (stable) continuous spectrum at some wave number, then become unstable over an interval of wave numbers, and in some cases eventually restabilize and merge back into the continuous spectrum. Such modes do not seem to have been observed in prior literature. The only results on single-phase instabilities in viscoelastic shear flow without streamline curvature that we are aware of are those of [25,26]; these papers, however, studied Poiseuille rather than Couette flow.

We also find unstable interfacial modes. We note that, for both the Johnson–Segalman and PEC-based models, the normal stress difference in the yielded phase is larger than in the unyielded