Journal of Non-Newtonian Fluid Mechanics 240 (2017) 15–33

Contents lists available at ScienceDirect

Journal of Non-Newtonian Fluid Mechanics

journal homepage: www.elsevier.com/locate/jnnfm

A high resolution spectral element approximation of viscoelastic flows in axisymmetric geometries using a DEVSS-G/DG formulation☆

R.M. Kynch, T.N. Phillips*

School of Mathematics, Cardiff University, Senghennydd Road, Cardiff CF24 4AG, United Kingdom

ARTICLE INFO

Article history:
Received 1 April 2016
Revised 22 December 2016
Accepted 28 December 2016
Available online 28 December 2016

Keywords:
Viscoelastic fluid
Flow past a sphere
Drag
Spectral element method
DEVSS-G
Discontinuous Galerkin

ABSTRACT

The discretisation of benchmark viscoelastic flow problems in axisymmetric geometries using the spectral element method is considered. The computations are stabilized using the DEVSS-G/DG formulation of the governing equations. A decoupled approach is employed in which the conservation equations are solved for velocity and pressure and the constitutive equation (Oldroyd-B and Giesekus) are solved for the polymeric component of the extra-stress tensor. The method is validated for the start-up of transient Poiseuille flow for which an analytical solution exists. A comprehensive set of results is presented for flow past a fixed sphere for the Oldroyd B and Giesekus models. Excellent agreement is found with results in the literature on the drag experienced by the sphere. Evidence is provided which shows the existence of a limiting Weissenberg number due to the inability to resolve the high gradients in axial stress in the wake of the sphere through polynomial enrichment. The shear-thinning property of the Giesekus model leads to a reduction in drag compared to the Oldroyd B model at equivalent values of the Weissenberg number and viscosity ratio. The numerical simulations eventually fail to converge for the Giesekus model which suggests that factors other than solely extensional properties are responsible for this behaviour. The influence of the Reynolds number and, for the Giesekus model, the mobility parameter on the drag coefficient is also investigated and discussed.

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1. Introduction

The computational simulation of flows involving viscoelastic fluids is a challenging task. The challenges present themselves in the choices of both the constitutive model for the fluid, and the numerical method used to approximate the solution in the chosen geometry. The choice of model must be made carefully, depending on the properties of the fluid and the dynamics of the flow which one wishes to simulate. The selected numerical method must be robust in terms of stability and accuracy. Few analytical solutions are available to validate the numerical method so it has become standard to use benchmark problems as a means of understanding the chosen model and validating the numerical scheme employed.

One established transient benchmark problem is that of a sphere of radius R_S falling at constant speed V_S inside a cylindrical tube [1]. This problem is one of the oldest problems in the study of fluid dynamics. It dates back to the work of Stokes in the mid 1800s [2] and has received continuing attention in the subsequent literature: a thorough history of the benchmark in the classical sense can be found in [3]. It is common to consider the problem in the frame of reference of the sphere and the walls, in the framework of the sphere, move upwards with uniform speed V_S and therefore in the opposite direction to the gravitational force.

In the context of viscoelastic flows, despite the simplistic nature of the geometry, this benchmark problem continues to present a challenging test for numerical schemes. The complex combination of shear and extensional flow regions and increasingly thin boundary layers has made consistent experimental and numerical results difficult to obtain [4]. The benchmark problem is also of practical interest in the context of flow around obstacles, for example in sedimentation, the settling of suspensions, rheometry and in industrial settings where particles are present (such as mineral and chemical processing or combustion engines).

It has become common that comparisons for this benchmark are made for the particular configuration in which the ratio of tube-to-sphere radius is 2 : 1 using the drag force, D computed on the surface of the sphere when the flow has reached a steady state. It is typical to make comparisons using the value of the

☆ All data for this research are openly available at http://doi.org/10.17035/d.2017.0031214755

* Corresponding author.
E-mail address: PhillipsTN@cf.ac.uk (T.N. Phillips).

http://dx.doi.org/10.1016/j.jnnfm.2016.12.008
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drag factor

$$D^* = \frac{D}{6\pi \eta_0 R_S V_S} \tag{1}$$

normalised using the drag experienced by the sphere in an unbounded expanse of Newtonian fluid with viscosity η_0. However, it has generally been agreed that, while useful, the drag factor does not provide enough insight into the global accuracy of the solution found by a numerical method [4,5], particularly as certain components of the stress do not feature in the calculation. It is therefore wise to provide further insight into the quality of the solution obtained using quantities such as the velocity and stress.

There are numerical results available for many models in the literature, with the upper-convected Maxwell (UCM) model featuring heavily in the literature (for example, Rasmussen and Hassager [6], Crochet and Legat [7], Baaijens et al. [8]). Other constitutive models considered include Oldroyd B, FENE-type, PTT and Giesekus [4]. The present work is focused on the Oldroyd B and Giesekus models.

Among the studies published on this benchmark using the Oldroyd B model (for example, see Lunsmann et al. [9], Bodart and Crochet [10], Tamaddon-Jahromi et al. [11]), only a few have used high-order methods (such as spectral or *hp*-finite element methods with high *p*), with most methods relying instead on very fine meshes (resulting in relatively high numbers of degrees of freedom) in order to show mesh convergence. Examples of higher-order methods applied to the problem are the spectral *p*-adaptive strategy of Chauvière and Owens [5] and the *hp*-adaptive finite element method of Fan [12] who together find agreement in the limiting Weissenberg number for this model. As with the UCM model there is a set of results which allows one to compile comprehensive tables of drag factors for comparison by other authors.

In the case of the Giesekus model, there have been many studies involving spheres, particularly in the investigation of experimentally observed phenomena (for example, Baaijens et al. [8], Yang and Khomami [13], Harlen [14]). However, there exist no definitive benchmark results available in the literature for the Giesekus model, at least in the sense that they are available for UCM and Oldroyd B models.

The aim of this paper is to apply a high-resolution spectral element method to the problem of benchmark of uniform flow past a fixed sphere for the Oldroyd B and Giesekus constitutive models, with model parameters commonly used by other authors. Our spectral element method is applied to a DEVSS-G/DG formulation of the problem to provide stabilisation. We shall present results which are convergent with respect to the spectral polynomial order, *p*, using a minimal number of elements. These results will add to those available in the literature for the Oldroyd B model and provide a reference for the Giesekus model, where few comprehensive results for this benchmark are available. A similar method, with a different implementation, has been successfully applied to the benchmark of flow past a cylinder [15] for these constitutive models and this paper will extend the available results with these techniques to the sphere benchmark problem.

This paper is arranged as follows. In Section 2 we describe the formulation of the governing equations including the DEVSS-G stabilisation and an alternative treatment of the continuity equation and also provide a brief discussion of the rheological behaviour of the constitutive models considered. In Section 3 we state the formulation of the sphere benchmark problem and how boundary conditions will be applied. Section 4 details the numerical methods applied to the temporal and spatial discretisations of the governing equations and how this is handled computationally. In Section 5 we present verification of our numerical scheme using the analytical solution for transient start-up of Poiseuille flow of an Oldroyd B fluid. This is followed by results for the sphere benchmark for the Oldroyd B and Giesekus models. Finally, in Section 6 we provide some concluding remarks.

2. Governing equations

Consider the Navier-Stokes equations in dimensionless form

$$Re\frac{D\mathbf{u}}{Dt} = -\nabla p + \beta\nabla^2\mathbf{u} + \nabla\cdot\boldsymbol{\tau} + \mathbf{f}, \tag{2}$$

$$\nabla\cdot\mathbf{u} = -\mu\int_\Omega p\,d\Omega, \tag{3}$$

where the field variables are velocity, $\mathbf{u}$, pressure, p, and elastic stress, $\boldsymbol{\tau}$, and $\mu > 0$ is a constant. The dimensionless groups are the Reynolds number, *Re*, and the viscosity ratio, β, which is the ratio of solvent to total viscosity.

The alternative statement of the continuity Eq. (3), proposed by Gwynllyw and Phillips [16] is a means of removing the indeterminacy in the pressure. It also ensures that when the weak statement of the problem is discretized, the pressure approximation is consistent with the choice of solution space, which requires that pressure possesses vanishing mean. There are also benefits to be gained in terms of the conditioning of the discrete problem albeit at the expense of a loss of sparsity in the global discrete system.

The system is closed by a constitutive law relating the elastic stress to the rate-of-deformation tensor, $\mathbf{d} = \frac{1}{2}\left(\nabla\mathbf{u} + \nabla\mathbf{u}^T\right)$. We consider the Giesekus [17] constitutive model for a viscoelastic fluid

$$\boldsymbol{\tau} + We\left(\overset{\triangledown}{\boldsymbol{\tau}} + \frac{\alpha}{(1-\beta)}\boldsymbol{\tau}^2\right) = 2(1-\beta)\mathbf{d} \tag{4}$$

where the dimensionless group *We* is the Weissenberg number and $\alpha > 0$ is the mobility parameter. We note that the Oldroyd B model [18] is a special case of (4) with $\alpha = 0$. We define the upper-convected derivative of a general tensor field, $\mathbf{G}$, by

$$\overset{\triangledown}{\mathbf{G}} = \frac{\partial\mathbf{G}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{G} - \mathbf{G}\cdot\nabla\mathbf{u} - (\nabla\mathbf{u})^T\cdot\mathbf{G}. \tag{5}$$

2.1. Model properties

The rheological properties of the constitutive models considered play an important role in terms of the type of behaviour that may be investigated using them. Two simple flows which provide insight into the model behaviour are uniaxial extension and simple shear, both of which are important mechanisms in flows involving spheres, with shear occurring near the surface of the sphere and extension occurring in the wake.

The Oldroyd model predicts an infinite extensional viscosity at a finite shear-rate (namely at $\dot{\epsilon} = \frac{1}{2We}$). This is an undesirable and unphysical property particularly when modelling flows involving extension. The Giesekus model does not suffer from this problem and predicts finite values at all extension rates with a limiting value [19] of $3\beta + 2\frac{(1-\beta)}{\alpha}$ for large extension rates. The Oldroyd B model predicts a constant shear viscosity whereas the Giesekus model predicts shear-thinning, with the rate of thinning with shear-rate increasing with the mobility parameter. The limiting behaviour of the Giesekus model is independent of the mobility parameter and tends to the solvent viscosity, i.e. β, for large shear-rates. The Oldroyd B model predicts a quadratic relationship between the first normal stress difference and shear-rate and a zero second normal stress difference. At low shear-rates the Giesekus model predicts a quadratic relationship between the first normal stress difference and shear-rate. However, this becomes linear at large shear-rates. The Giesekus model predicts a non-zero second normal stress difference, which tends to the value $-\frac{(1-\beta)}{We}$ with increasing shear-rate.

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