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The dam-break problem for eroding viscoplastic fluids

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A B S T R A C T

Natural gravity-driven flows can increase in volume by eroding the bed on which they descend. This process is called basal entrainment and is thought to play a key role in the bulk dynamics of geophysical flows. Although its study is difficult using field measurements, basal entrainment is more easily amenable to analysis using laboratory experiments. We studied basal entrainment by conducting dam-break experiments releasing a fixed amount of viscoplastic fluid (a Herschel–Bulkley fluid) on a sloping, erodible bed of fixed depth. Entrainment was observed continuously, far from the sidewalls, using cameras. Bed material was quickly entrained, which led to flow advancement. Although the slope inclination had clear effects on the entrainment mechanisms, as shown by the internal measurements, this did not translate into faster front progression. Instead, the depth and length of the entrainable material were the most important controlling parameters of front velocity, as the surge scoured out the entrainable layer, pushing the entrainable material downstream and following the rigid bed's geometry. Bulk measurements (front position and flow depth profile) were also compared with predictions from lubrication theory.

1. Introduction

Gravity driven flows, such as snow avalanches and debris flows, pose a threat to human activities and settlements in mountain areas. The economic importance of these activities (e.g. mining, forestry, electricity production, tourism, transportation) has encouraged research into methods for calculating the main features of these flows (e.g. run-out distance, flow depth, impact force) [1]. In the 1960s, the idea emerged that an analogy could be made between avalanches and water flows, and since then the Saint–Venant equations have been increasingly used to describe the motion of “snow floods” [2–4], rock avalanches [5], debris flows [6], turbidity currents [7], and submarine avalanches [8].

Although the analogy with water waves has been pivotal to laying out the mass and momentum balance equations, there are crucial differences between water and natural materials involving mixtures of fluids and solids. A large amount of research has been done to determine the effects of bulk composition on rheological behaviour, flow resistance and self-organisation during flow. Another key difference between water and natural materials is related to mass exchanges between the flow and the bed: gravity driven flows can grow in size by mobilising loose material lying in their paths, or they can lose mass as a result of various processes (e.g. levee formation, debulking due to solid particle sedimentation) [9–15]. This raised the question of whether basal entrainment affects bulk dynamics. For instance, for powder-snow avalanches, Kulikovskiy and Svehnikova [16] developed a simple model which took into account the incorporation of air and snow and showed that basal entrainment plays an essential part in the growth of high-velocity avalanches. Without snow entrainment, air entrainment causes a dilution of the snow cloud, and thus a decrease in buoyancy forces [17]. Generalising the depth-averaged Saint–Venant equations to eroding flows mobilising natural materials has proved challenging to the different groups working on the issue. Recently, Iverson and Ouyang [18] reviewed the various attempts to model mass exchanges between flows and beds within the framework of the Saint–Venant equations. They showed that many existing models violated mass and momentum conservation laws, mostly because the boundary conditions at the bed-flow interface were incorrect. One underlying issue raised by their review was the absence of closure equations for the entrainment and deposition rates.

To shed light on basal entrainment's effects on the behaviour of gravity-driven flows, we investigate a problem that retains the essential features of natural scenarios, while being sufficiently simple to be manageable semi-analytically. We consider the dam-break problem for a viscoplastic fluid, i.e. the flow of a fixed volume of fluid suddenly released down a slope from a reservoir. The sloping bed is a solid substrate, but at a certain distance from the reservoir, the flow enters into contact with an erodible stationary layer composed of the same fluid and starts entraining it. We sought to

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determine how basal entrainment affected the front position and flow-depth profile over time.

The viscoplastic dam-break problem is a typical example of time-dependent flow, in which the flow passes through different phases from release to run-out. This problem has been studied within the frameworks of the Saint–Venant equations [19–22] and lubrication theory [23–30]. Based on the assumptions that the flow is shallow (i.e. the aspect ratio $\epsilon = L/H$, where L and H denote scales of length and depth) and slow (i.e. the Reynolds flow number is low), lubrication theory approximates the local momentum-balance equation using an asymptotic expansion in ϵ. The decisive advantage of lubrication theory over the Saint–Venant equations is that the velocity and stress fields are calculated with no recourse to closure equations as long as inertia plays a negligible role.

In the present paper, we focus on a nonlinear class of viscoplastic materials called Herschel–Bulkley fluids. Viscoplastic materials behave like fluids when they are sufficiently stressed, but like solids when the stress state is below a given threshold (called the yield stress) [31–34]. As natural materials exhibit solid- and fluid-like properties, the use of viscoplastic models has been proposed in order to describe the rheological behaviour of snow [35], mud [20], debris mixtures [36–38], lava [39] and submarine mud [8]. Actual rheological behaviour exhibits complex properties—such as two-phase effects (pore pressure diffusion), dilatancy, particle migration and segregation [6,40–42]—which are not accounted for by the simple constitutive equations of single-phase continua such as the Herschel–Bulkley equation. Yet in spite of these limitations, the Herschel–Bulkley equation provides a useful approximation of various natural viscoplastic flows [20,22,24,27,28,43–47]. As viscoplastic models deal with the solid-liquid transition, they also seem relevant for describing basal entrainment: part of the bed may yield under the effects of the normal and shear stresses exerted by the flow, and then be entrained in that flow. This is, for instance, what is thought to happen in snow avalanches [48–50].

In this paper, we tackle the issue of basal entrainment using lubrication theory. We begin with a theoretical perspective of basal entrainment in shallow flows within the framework of lubrication theory (see Section 2). In Section 3, we describe the experimental procedure used for measuring the flow variables and observing what happens inside eroding flows. Section 4 presents our experimental results and compares them with theoretical predictions from lubrication theory. Section 5 concludes the paper. Three videos are available to accompany this paper (the acknowledgements section provides the link to the data repository).

2. Dam-break wave eroding a stationary layer

This section examines the effects of basal entrainment on the front motion of a viscoplastic avalanche. Let us consider that at time $t = 0$, an avalanche made up of a Herschel–Bulkley fluid is released from a reservoir. Initially the fluid material flows over a sloping solid boundary. The bottom inclination is denoted by θ. At time $t = t_0$, the material encounters a stationary layer made up of the same fluid and occupying a step of length ℓ_{bed} (see Fig. 1). The viscoplastic flow spreads across this stationary layer and entrains part of it. The front position is denoted by $x_f(t)$, the flow depth by $h(x, t)$ and the velocity field by $\boldsymbol{u} = (u, w)$. We use a Cartesian frame with the x-axis pointing downward and the z-axis normal to the slope.

To solve this problem, we use lubrication theory. Within the framework of this theory, the momentum balance equations are simplified by neglecting inertia terms and the streamwise gradient of the normal stress. This makes it possible to deduce the pressure and shear stress distributions to the leading order. Making use of the constitutive equation then leads to the velocity profile and, finally, the depth-averaged mass conservation provides the evolution equation for the flow depth $h(x, t)$. There is a large body of work applying this theory to viscoplastic flows [23,26,27,30,51]; it is succinctly summarised in the next section.

2.1. Solution for rigid bottoms

In the limit of low Reynolds number and small aspect ratio numbers, motion is dictated by the balance between the streamwise gradient of the pressure $\partial_x p$, gravitational forces and the cross-stream gradient of the shear stress $\partial_y \tau$. To the first order, the pressure p adopts a hydrostatic distribution, while the shear stress τ follows a linear distribution whose coefficient is controlled by the bed slope and free surface gradient:

$$p = \varrho g(h-z)\cos\theta \text{ and } \tau = \varrho g \sin\theta (h-z)\left(1 - \cos\theta \frac{\partial h}{\partial x}\right). \quad (1)$$

These expressions hold regardless of the constitutive equation.

We now consider the constitutive equation for simple Herschel–Bulkley materials

$$\begin{cases} \dot{\gamma} = 0 & \text{if } \tau < \tau_c, \\ \tau = \tau_c + \kappa|\dot{\gamma}|^n & \text{if } \tau \geq \tau_c, \end{cases} \quad (2)$$

where τ_c denotes the yield stress, $\dot{\gamma} = \mathrm{d}u/\mathrm{d}z$ the shear rate, n the shear-thinning index (as in most cases $n \leq 1$) and κ the consistency. These materials flow when the basal-shear stress exceeds the yield stress τ_c. When this condition is satisfied, there exists a surface $z = Y(x, t)$ where the shear stress equals the yield stress:

$$Y = h - \frac{\tau_c}{\varrho g \sin\theta \left|1 - \cot\theta \frac{\partial h}{\partial x}\right|}. \quad (3)$$

Below this surface, the fluid is sheared and above this surface it moves like a plug. Equations (2) and (1) lead to the following expression for the streamwise velocity component $u(x, z, t)$

$$u(x,z,t) = \frac{n}{n+1} A \left(1 - S\frac{\partial h}{\partial x}\right)^{1/n} \left(Y^{1+1/n} - (Y-z)^{1+1/n}\right) \quad \text{for} \quad 0 \leq z \leq Y, \quad (4)$$

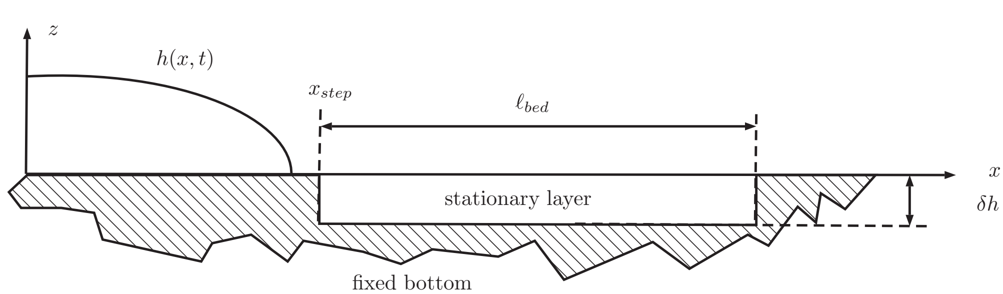

Fig. 1. Configuration of the flow. A viscoplastic avalanche is released from a reservoir. It flows over a sloping rigid bed until it gets in contact with a stationary layer made of the same fluid.