Journal of Non-Newtonian Fluid Mechanics 247 (2017) 22–40

Contents lists available at ScienceDirect

Journal of Non-Newtonian Fluid Mechanics

journal homepage: www.elsevier.com/locate/jnnfm

Buoyancy effects on micro-annulus formation: Density stable displacement of Newtonian–Bingham fluids

M. Zare[a,*], A. Roustaei[b], I.A. Frigaard[c]

[a] Department of Mechanical Engineering, University of British Columbia, 2054-6250 Applied Science Lane, Vancouver, BC, V6T 1Z4, Canada
[b] Department of Engineering Science, College of Engineering, University of Tehran, Tehran, 11155-4563, Iran
[c] Departments of Mathematics and Mechanical Engineering, University of British Columbia, 1984 Mathematics Road, Vancouver, BC, V6T 1Z2, Canada

ARTICLE INFO

Article history:
Received 15 February 2017
Revised 2 June 2017
Accepted 10 June 2017
Available online 15 June 2017

Keywords:
Buoyant displacement
Bingham fluid
Residual wall layer
Flow regimes

ABSTRACT

Buoyant miscible displacement flow of a Bingham fluid by a Newtonian fluid along a vertical plane-channel is studied, in the high Péclet number regime. The displacing fluid is denser than the displaced fluid and the flow direction is density-stable (upwards). The flow is effectively governed by 4 dimensionless parameters: the Newtonian Bingham number (B_N), the viscosity ratio (m), the Reynolds number (Re), and modified Froude number (Fr). This is a simple model for micro-annulus formation in the primary cementing of oil and gas wells. We show that the residual layer thickness is largely determined by two parameters: (B_N/m, χ^*/m), where $\chi^* = \frac{2Re}{Fr^2}$. Residual wall layers may be either static or mobile, and mobile layers either evolve to become static or are washed from the channel at long times. We show that the different behaviours of the residual layers are linked to different characteristic behaviours of the displacement front, and we show how the latter behaviours can be predicted using a lubrication/thin-film approximation.

1. Introduction

This paper studies miscible displacement flows in long vertical channels, in which the displaced fluid has a yield stress and the displacing fluid does not, i.e. the configuration is density stable. As is common in such flows, the initial displacement is incomplete (not fully efficient) and a residual layer of the in-situ fluid can be left behind on the walls. In the case that the displaced fluid is purely viscous the residual layers drain over time or mix. However, if the residual fluid has a yield stress, residual layers may remain indefinitely; see [1]. The aim of the paper is to predict the long-time behaviour of residual layers (as $t \to \infty$), in terms of the dimensionless groups of the flow. In particular we are interested in the effects of the stable buoyancy gradient on the flow, which differentiates this study from the iso-density flows studied in [1–3].

The chief motivation behind our study is the operation of primary cementing, which occurs when oil and gas wells are constructed; see [4]. The main objective of primary cementing is to provide complete and permanent isolation of different fluid-bearing rock formations by sealing the drilled hole hydraulically with cement. During this process, a steel casing is inserted into the newly drilled section of the well. The in situ drilling mud must be fully replaced with cement slurry between the casing and the formation. To this end, a number of fluids (e.g. wash, then spacer, then cement slurry) are pumped down the inside of the casing and return upwards through the narrow annulus between the outside wall of the steel casing and the inside wall of the surrounding rock formation, see Fig. 1a. Drilling muds typically have a yield stress, which is important during drilling for cuttings transport. The displacing fluids (wash, spacer and eventually the slurry) have varied rheology.

The yield stress in the drilling mud allows it to resist the imposed stresses during displacement and hence to remain static in the annulus, attached to the walls. The existence of thin residual mud layers has been termed a (fluid-filled or wet) micro-annulus. Cemented annuli are rarely concentric and residual layers tend to be thicker on the narrow side of the annulus where the shear stresses are reduced. In extreme cases, drilling mud layers can bridge between the annulus walls to form a mud channel, usually at the narrowest point [5], although this process is not fully understood [6]. Wet micro-annuli dehydrate during setting of the cement, (dry micro-annuli may also form due to shrinkage), and these layers provide the possibility for gas (or liquids) from one geological zone to hydraulically connect to another zone, i.e. it provides both entry points for fluid invasion and paths along which

---

[*] Corresponding author.
E-mail addresses: marjan.zare@math.ubc.ca (M. Zare), frigaard@math.ubc.ca (I.A. Frigaard).

http://dx.doi.org/10.1016/j.jnnfm.2017.06.002

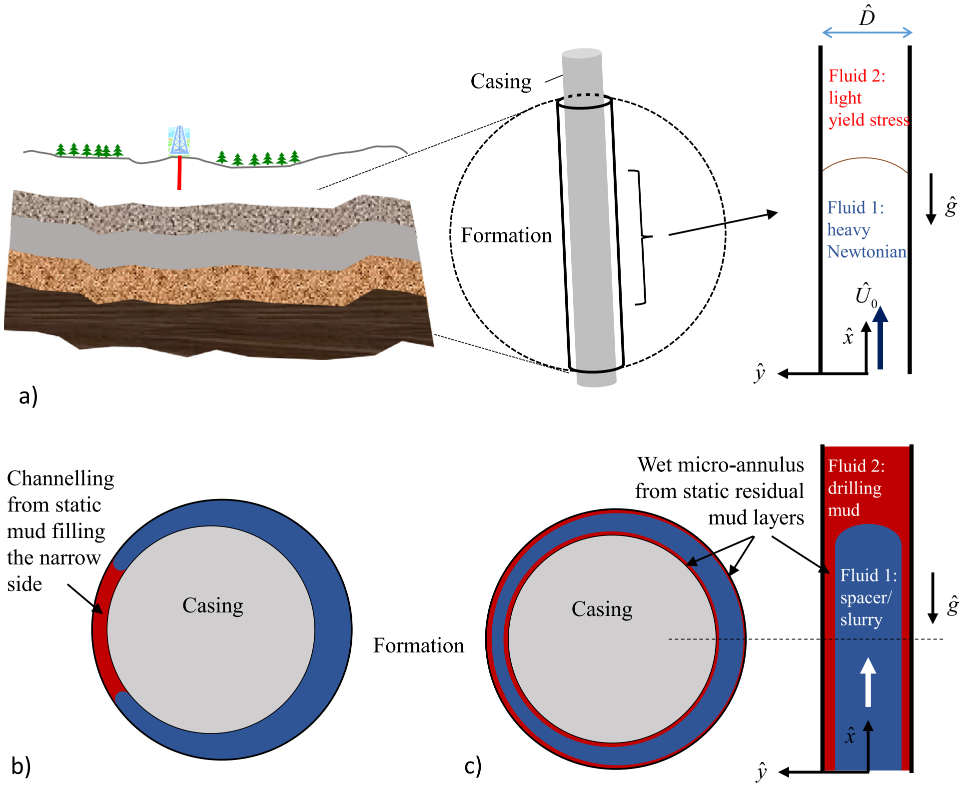

Fig. 1. Primary cementing of an oil well. a) Fluid-fluid displacement in a narrow eccentric annulus modelled as a longitudinal channel. b) Channelling of mud due to eccentricity, illustrated in a cross-section e.g. [5]. c) Formation of a fluid-filled micro-annulus (modelled here).

fluid can migrate. See Fig. 1b & c for an illustration of these fluid flow-related defects.

Here we are concerned with the initial formation of micro-annuli during displacement. Cemented annuli are a few centimeters wide and 100's of meters long. The fluids involved are typically miscible, but due to the geometry and flow rates, Péclet numbers (Pe) are very large. Mathematically, we study the limit $Pe \to \infty$ of (laminar) miscible displacement flows. An equivalent limit is that of immiscible displacement flows with infinite Capillary number Ca. Thus, we draw insight from both miscible and immiscible displacement flows.

The study of immiscible displacements in capillary tubes was initiated more than 50 years ago by Taylor [7], and many studies have followed. For two viscous liquids the residual layer thickness asymptotes to a constant value as $Ca \to \infty$. At fixed Ca the residual layer thickness interestingly decreases as the viscosity ratio of displaced to displacing fluid increases (denoted m in our study) see [8], which is counter-intuitive, i.e. the thinnest residual layers are found for gas-liquid displacement. In the miscible fluid setting [9,12] studied displacements of a viscous fluid with a less viscous fluid in capillary tubes computationally and experimentally, respectively. It was observed that at large $Pe (\gtrsim 10^5)$ the residual layer thickness also asymptotes to a constant value but that this layer thickness increases with viscosity ratio m. Some comparison is made with the large Ca limit, although it is pointed out that the original results of [7] do not extend to this limit. This apparent discrepancy has been discussed by [10], who attribute the discrepancy to the fact that the residual layers are not stationary for comparable viscosity fluids, which affects the calculation of residual layer thickness in [9] via the method of [7]. This is shown more clearly in the recent experiments of [11].

Buoyancy effects were included in [9], but these were not particularly significant at large Pe. Buoyancy was studied in more detail by Lajeunesse et al. [13], who studied density stable displacements of miscible fluids in a Hele-Shaw cell, both experimentally and theoretically. Flows were studied over a range of viscosity ratios and buoyancy numbers, and not strictly confined to low Re. We return to [13] in analyzing the flow types observed in our numerical simulations.

Whereas Newtonian fluids continuously deform when a shear stress is applied, yield stress fluids do not have to deform. Therefore, residual layers of yield stress fluid can remain forever. Allouche, et al. [1] studied the miscible displacement of two visco-plastic fluids in a plane channel numerically and determined sufficient conditions for the non-existence of a static wall layer using a simple 1D model. Gabard and Hulin [2] experimentally studied miscible displacements of non-Newtonian fluids with zero and non-zero yield stresses by less viscous and mostly Newtonian fluids of the same density in a vertical tube. These were generally at low-moderate Re and for yield stress displaced fluids; the experiments showed a steadily moving front leaving behind a uniform thickness static layer, qualitatively analogous to the flows in [1]. In [1] predictions were made of the static layer thickness that represented the computed layers thickness reasonably well and depended primarily on the downstream fluid flow. However, deeper examination in [3] showed that the predictions of [1] could not account for observed variations in layer thickness as Re was increased, and predicting the layer thickness remains unsolved.

Here we revisit this problem and also consider the effects of a positive density difference to aid in displacing the fluids. This is indeed usual in primary cementing for laminar flows as the displacing fluids are usually pumped in a sequence of increasing density (mud, spacer, cement slurry); see e.g. [4]. Due to the large number of dimensionless parameters involved in displacement of two shear-thinning yield stress fluids with yield stress in a 3D annulus, we simplify both geometrically and rheologically in considering a