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Simulation of velocity and shear stress distributions in granular column collapses by a mesh-free method

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ABSTRACT

To describe granular flows in the dense regime, the $\mu(I)$ rheology model was proposed. It has been proven to be effective in reproducing flow dynamics in the dense regime caused by dry granular materials. For continuum modeling, the mesh-free method can easily handle flows with interface such as free surface, which commonly exists in most granular flows. In this study, the $\mu(I)$ rheology model is coupled with a mesh-free method, Moving Particle Semi-implicit method (MPS). The coupled model is used to analyze velocity and shear stress distributions in granular column collapses. To validate the model, velocity measurements were conducted on two aspect ratios $a = 1.25$ and 5.0. The coupled model is verified by the measured velocity profiles. Both horizontal and vertical velocity distributions are examined in the validation. A linear relationship on the velocity distribution is observed in the flowing region in the collapses both experimentally and numerically. Another larger aspect ratio $a = 7.0$ were then simulated and a similar linear velocity distribution was obtained. On the basis of the velocity analysis, the tangential shear stress was analyzed and discussed in the three collapses. It showed that the distribution of the shear stress is symmetrical with the opposite direction. In the core quasi-static region, the shear stress was larger than that in the flowing region. In the free fall of the upper portion for the large aspect ratios such as $a = 7.0$, there was very small shear stress. In the center of the column in the collapses, the shear stress almost remains zero with some fluctuations.

1. Introduction

Granular materials are very common in diverse fields such as geotechnical engineering, agriculture, powder technology, and the pharmaceutical industry. They are able to behave differently under different circumstances [1–3]. Resembling like a liquid, they flow through channels or pipes, or run as surface waves, etc. In other situations, they can behave more like a solid. In regards to dry and cohesion-less granular materials, behaviors of them are also complicated [4–9,38,40,41]. The flows caused by these materials are usually divided into three regimes based on the velocity [10–12]: the quasi-static regime, where the grain inertia is negligible and flow is often described by soil plasticity models [13]; an intermediate dense regime; and the gaseous regime if the material is strongly agitated. In the gaseous regime, the grains are far apart with each other and the interaction is dominated by binary collisions. To account for this behavior, a kinetic theory has been developed [14]. The dense regime is a regime where grain inertia is significant and there is a contact network among grains. One method to predict the granular flow in the dense regime is to use a continuum description which requires a constitutive equation. Advances have been made in developing a constitutive law for the dry granular flows as the $\mu(I)$ rheology model [10–12].

The application of this rheology model in the continuum approach has been proven to be able to reflect many characteristics in granular flows [15–19]. The $\mu(I)$ rheology model is validated in the finite volume method with the use of volume-of-fluid (VOF) technique for tracking free surface by Lagrée et al. [16]. The flow dynamics in granular column collapses is successfully obtained in their research. With the $\mu(I)$ rheology model, a three-dimensional simulation study has been conducted by using the finite element method [19]. A regularization technique for the rheology model is shown to be effective in reproducing flow dynamics. In the simulations by Ionescu et al. [15], the rheology model in the finite element method is examined with respect to rheological parameters and boundary conditions. It is shown that the rheology model in the numerical method is good at quantitatively reflecting experimental results in the granular column collapse over inclined planes. The $\mu(I)$ rheology model is also implemented in mesh-

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free methods such as Moving Particle Semi-implicit method (MPS) [18] and Smoothed Particle Hydrodynamic (SPH) [17,51]. It is unnecessary to employ extra techniques such as VOF in these methods in handling free surface. The flow characteristics such as free surface and wave front are reported to be captured in the meshfree methods [17,18]. Besides, there are many studies by using the $\mu(I)$ rheology model in the depth-integrated equations [20–22]. Although many flow dynamics have been reproduced, velocity and shear stress calculated by using the $\mu(I)$ rheology model in the numerical methods need to be addressed in granular flows.

MPS as a Lagrangian method is a good numerical tool in simulating free-surface flows. It was first proposed by Koshizuka and Oka [23] and then widely used in various engineering areas such as coastal engineering [24–26], nuclear engineering [27], hydraulic engineering [29–31], biomechanics [28], etc. To simulate dry granular flows without cohesion, MPS is coupled with the $\mu(I)$ rheology model. The coupled model is assessed on calculating velocity and shear stress in an unsteady configuration, namely, granular column collapse. The flow is activated by instantaneously releasing a column of granular medium such as glass beads and then a highly unsteady flow with varying free surface develops. Many experiments have been conducted to study flow characteristics in these column collapses [32–37]. It is found that an initial aspect ratio plays an important role on the spreading of the column (e.g. [33,36]). The free-surface profiles and wave fronts in the column collapses are reported to be successfully captured by the coupled model [18]. In this study, the velocity distribution in the collapses is analyzed. To achieve this, experiments were conducted to measure the velocity in the flows. Velocity from different initial aspect ratios, which can trigger different flow characteristics, are measured. The coupled model is validated by the experimental results and the velocity distribution for different aspect ratios is analyzed both numerically and experimentally. Based on the velocity analysis, the ability of the coupled model in analyzing shear stress is examined and discussed.

The paper is organized as follows: the $\mu(I)$ rheology model and the governing equations are in Section 2; Section 3 illustrates methodology used in the current research, including the numerical method and experimental setup; Sections 4–6 are result discussions of flow dynamics, velocity distribution, and shear stress in the collapses, respectively.

2. Theoretical framework

2.1. The inertial number I

In the intermediate regime, the granular material is in close contact with one another, yielding a contact network among them. Grains interact both by enduring contact and collisions; thereby inertia plays an important role in their behaviors. Due to complex correlations of motion and force, theoretical description of such flows is very challenging beyond the kinetic theory in the gaseous regime [14] and also beyond the soil plasticity models in the quasi-static regime [13]. Advances to understand the flows in the dense regime have been made in the last decades through experimental and numerical research in various configurations. A detailed review can be found in the literature [10].

To quantify the relative importance between inertial and confining stresses, a dimensionless number, inertial number I, is introduced [10]:

$$I = |\gamma| d_s \sqrt{\rho_s / p}. \tag{1}$$

where d_s is the grain size in diameter, ρ_s is the density of granular material, and p is the pressure. In Eq. (1), $|\gamma|$ represents the second invariant of the strain rate tensor,

$$|\gamma| = \sqrt{0.5\gamma_{\alpha\beta}\gamma_{\alpha\beta}} \quad \text{with} \quad \gamma_{\alpha\beta} = \frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial u_\alpha}. \tag{2}$$

where $\gamma_{\alpha\beta}$ is the strain rate tensor, and u_α, u_β are velocity components in the α- and β- direction, respectively.

2.2. The μ(I) rheology model

The pioneering work on the granular flows was made by Bagnold [39]. However, to predict granular flows in the dense regime, the simple rheology models such as Newtonian, Bingham, and Bagnold were unable to simulate good results in granular column collapses [16]. There are also some models developed from different theoretical backgrounds, but they can reproduce limited behaviors in granular flows [54–56]. The Drucker–Prager and Mohr–Coulomb models are able to calculate the plastic deformation caused by granular materials [57,58]. As a more recently developed model, the $\mu(I)$ rheology model is successful in reproducing dynamics in the dense regime of dry granular flows in different configurations. The $\mu(I)$ model was developed from quantifying observations in a series of experiments and numerical simulations (e.g. [10,11]). The effectiveness of the $\mu(I)$ rheology model can range in a fairly wide class of granular flows in the dense regime [11,42,52]. The model and its extensions have been applied into modeling dry granular flows in different numerical method [6,15–18,52,59], successfully reproducing flow dynamics such as run-off, free surface, and flow patterns in the granular column collapses. Therefore, the $\mu(I)$ rheology model is selected in this study to investigate velocity and stress in granular column collapses.

When the granular flow develops, the local tangential shear stress τ is proposed to be analogous to the classical Mohr-Coulomb friction law in practice, i.e. whose value is proportional to the local normal pressure p,

$$\tau = \mu p. \tag{3}$$

where μ is the friction coefficient, which is locally dependent on the inertial number [11]:

$$\mu(I) = \mu_s + \frac{\mu_2 - \mu_s}{I/I_0 + 1}. \tag{4}$$

Eq. (4) includes two material-dependent coefficients μ_s and μ_2, where μ_s denotes the threshold value for the quasi-static regime ($I \to 0$) and $\mu(I)$ converges to a limiting value μ_2 for high I ($I >> 1$) in the strongly sheared regime. For spherical glass beads, $\mu_s = \tan(20.9°)$, $\mu_2 = \tan(32.76°)$, and $I_0 = 0.279$ are suggested [11], which are used in this study.

Although grains are in dense contact in the intermediate regime, a small variation in the volume fraction ϕ is still observed, where the inertial number may play an important role. With increase of the inertial number, the volume fraction decreases because the grains are agitated and can slightly separate. Letting ϕ_{max} be the maximum volume fraction in the system, the volume fraction ϕ is given by [12] and [43]:

$$\phi = \phi_{\max} - \delta_\phi I. \tag{5}$$

where δ_ϕ is 0.2 [12].

In the case of neglecting the small variation in the volume fraction and assuming the fluid is incompressible, the internal stress tensor $\tau_{\alpha\beta}$ is proposed [11]:

$$\tau_{\alpha\beta} = \eta(|\gamma|, p)\gamma_{\alpha\beta}, \tag{6}$$

$$\eta(|\gamma|, p) = \mu(I)p/|\gamma|. \tag{7}$$

where η is the effective viscosity.