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Two–dimensional viscoplastic dambreaks

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ABSTRACT

We report the results of computations for two–dimensional dambreaks of viscoplastic fluid, focusing on the phenomenology of the collapse, the mode of initial failure, and the final shape of the slump. The volume-of-fluid method is used to evolve the surface of the viscoplastic fluid, and its rheology is captured by either regularizing the viscosity or using an augmented-Lagrangian scheme. We outline a modification to the volume-of-fluid scheme that eliminates resolution problems associated with the no-slip condition applied on the underlying surface. We establish that the regularized and augmented-Lagrangian methods yield comparable results, except for the stress field at the initiation or termination of motion. The numerical results are compared with asymptotic theories valid for relatively shallow or vertically slender flow, with a series of previously reported experiments, and with predictions based on plasticity theory.

1. Introduction

The sudden gravitational collapse of a mass of viscoplastic fluid features in a diverse range of problems from geophysics to engineering. These flows can constitute natural or manmade hazards, as in the disasters caused by mud surges and the collapse of mine tailing deposits. In an industrial setting, the controlled release of a reservoir in a simple dambreak experiment forms the basis of a number of practical rheometers, including the slump test for concrete [1,2] and the Bostwick consistometer of food science [3]. The slump test features the release of a cylinder of yield-stress fluid. The focus of the current article is more aligned with the Bostwick consistometer, in which materials such as ketchup are released in a rectangular channel, and two–dimensional flow is a convenient idealization. In view of the relatively slow nature of the flows in many of these problems, we also consider the limit of small inertia.

Despite wide–ranging practical application, the theoretical modelling of viscoplastic dambreaks remains relatively unexplored. Asymptotic theories for shallow, slow flow have received previous attention and permit a degree of analytical insight into the problem (see [4,5] and references therein). Numerical computations of two–dimensional dambreaks have also been conducted to model flows that are not necessarily shallow [6]. However, these simulations do not provide a detailed survey of the flow dynamics over a wide range of physical conditions and have focused mainly on determining some of the more qualitative aspects of the end state of a slump, such as its final runout and maximum depth. Complementing both asymptotics and numerical simulation are cruder predictions of the final shape based on solid mechanics and initial failure criteria derived from plasticity theory [1,7,8].

The key feature of a viscoplastic fluid that sets the problem apart from a classical viscous dambreak is the yield stress. When sufficient, this stress can hold the fluid up against gravity, preventing any flow whatsoever. If collapse does occur, the yield stress brings the fluid to a final rest and can maintain localized rigid regions, or "plugs", during the slump. The evolving plugs and their bordering yield surfaces present the main difficulty in theoretical models, particularly in numerical approaches. Augmented-Lagrangian schemes that deal with the complications of the yield stress directly are often time-consuming to run, whereas regularizations of the constitutive law that avoid true yield surfaces introduce their own issues [9]. For the dambreak problem, difficulties are compounded by the need to evolve the fluid surface and impose boundary conditions such as no-slip on the substrate underneath the fluid.

In the current paper, we present numerical computations of viscoplastic dambreaks spanning a wide range of physical parameters. Our aim is to describe more fully the phenomenology of the collapse and its plugs, the form of the motion at initiation, and the detailed final shape. Our main interest is in the effect of the yield stress, so we consider Bingham fluid, ignoring any rate-dependence

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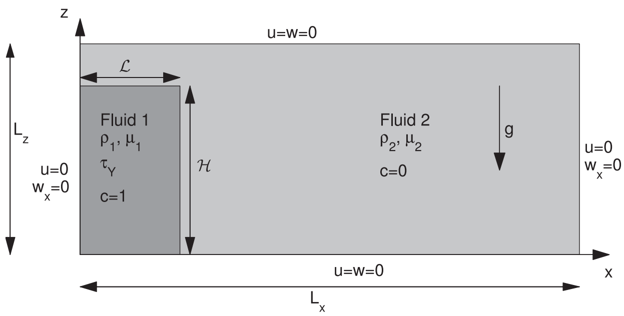

Fig. 1. A sketch of the geometry for the case of a rectangular initial block.

of the plastic viscosity. We mathematically formulate the dambreak problem in Section 2 and outline the numerical strategies we use for its solution. We use both an augmented-Lagrangian scheme and regularization of the constitutive law to account for viscoplasticity; to deal with the free surface, we use the volume-of-fluid method. The latter method emplaces the viscoplastic fluid beneath a less dense and viscous fluid, then tracks the interface between the two using a concentration field. This effectively replaces the single-phase dambreak problem with that of a two-phase miscible fluid displacement (we ignore surface tension), but introduces a significant complication when imposing a no–slip boundary condition: because the lighter fluid cannot be displaced from the lower surface, the slumping heavier fluid over–rides a shallow finger of lighter fluid which lubricates the overlying flow and thins continually, leading to difficulties with resolution. We expose this complication for a viscous test case in Section 3, and identify means to avoid it. We then move on to a discussion of Bingham dambreaks in Section 4, before concluding in Section 5. The appendices contain additional technical details of the numerical schemes, asymptotic theories for shallow or slender flow, and some related plasticity solutions.

2. Formulation

2.1. Dambreak arrangement and solution strategy

To simulate the collapse of a Bingham fluid, we consider a two–fluid arrangement, with the yield-stress fluid emplaced underneath a lighter viscous fluid. We ignore any interfacial tension. The volume-of-fluid method is used to deal with the boundary between the two fluids: a concentration field $c(x, y, t)$ smooths out and tracks the fluid-fluid interface; $c = 1$ represents the viscoplastic fluid and the overlying Newtonian fluid has $c = 0$. The concentration field satisfies the advection equation for a passive scalar; no explicit diffusion is included although some is unavoidable as a result of numerical imprecision. Fig. 1 shows a sketch of the geometry; the initial block of viscoplastic fluid has a characteristic height $\mathcal{H}$ and basal width $2\mathcal{L}$, but we assume that the flow remains symmetrical about the block's midline and consider only half of the spatial domain.

To deal with the yield stress of the viscoplastic fluid, we use both an augmented-Lagrangian scheme [10] and a regularization of the Bingham model. The numerical algorithm is implemented in C++ as an application of PELICANS[1]. We refer the reader to [11,12] for a more detailed description of the numerical method and its implementation. We use the regularized scheme as the main computational tool; the augmented–Lagrangian algorithm is slower and was used more sparingly, specifically when looking at flow close to failure or during the final approach to rest. In most situations, the agreement between the two computations is satisfying (examples are given below in Fig. 4); only at the initiation or cessation of motion is there a noticeable difference, primarily in the stress field (discounting the solution for the plug, which is an artifact of the iteration algorithm in the augmented–Lagrangian scheme).

2.2. Model equations

We quote conservation of mass, concentration and momentum for a two-dimensional incompressible fluid in dimensionless form:

$$\nabla \cdot \boldsymbol{u} = 0, \qquad \frac{\partial c}{\partial t} + (\boldsymbol{u} \cdot \nabla) c = 0, \tag{1}$$

$$\rho Re \left[\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla) \boldsymbol{u} \right] = -\nabla p + \nabla \cdot \boldsymbol{\tau} - \rho \hat{\boldsymbol{z}}, \tag{2}$$

In these equations, lengths $\boldsymbol{x} = (x, z)$ are scaled by the characteristic initial height of the Bingham fluid, $\mathcal{H}$, velocities $\boldsymbol{u} = (u, w)$ by the speed scale $\mathcal{U} = \rho_1 g \mathcal{H}^2/\mu_1$, and time t by $\mathcal{H}/\mathcal{U}$, where g is the gravitational acceleration; the stresses $\boldsymbol{\tau}$ and pressure p are scaled by $\rho_1 g \mathcal{H}$. The Reynolds number is defined as $Re = \rho_1 \mathcal{U}\mathcal{H}/\mu_1$. Here, the subscript 1 or 2 on the (plastic) viscosity μ and density ρ distinguishes the two fluids, and linear interpolation with the concentration field c is used to reconstruct those quantities for the mixture; *i.e.* after scaling with the denser fluid properties,

$$\rho = c + (1 - c)\frac{\rho_2}{\rho_1} \qquad \text{and} \qquad \mu = c + (1 - c)\frac{\mu_2}{\mu_1}. \tag{3}$$

In dimensionless form, the unregularized Bingham constitutive law is

$$\begin{cases} \dot{\gamma}_{jk} = 0, & \tau < cB, \\ \tau_{jk} = \left(\mu + \dfrac{cB}{\dot{\gamma}} \right) \dot{\gamma}_{jk}, & \tau > cB, \end{cases} \qquad \tau = \sqrt{\frac{1}{2} \sum_{j,k} \tau_{jk}^2} \tag{4}$$

where

$$B = \frac{\tau_Y \mathcal{H}}{\mu_1 \mathcal{U}} \equiv \frac{\tau_Y}{\rho_1 g \mathcal{H}} \tag{5}$$

is a dimensionless parameter related to the yield stress τ_Y, and the deformation rates are given by

$$\dot{\gamma}_{jk} = \frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j}, \qquad \dot{\gamma} = \sqrt{\frac{1}{2} \sum_{j,k} \dot{\gamma}_{jk}^2}. \tag{6}$$

The regularized version that we employ is

$$\tau_{jk} = \left(\mu + \frac{cB}{\dot{\gamma} + \epsilon} \right) \dot{\gamma}_{jk}, \tag{7}$$

where ϵ is a small regularization parameter. We verified that the size of this parameter had no discernible effect on the results presented below; we therefore consider irrelevant the precise form of the regularization (which is simple, but not necessarily optimal).

We solve these equations over the domain $0 \le x \le \ell_x = L_x/\mathcal{H}$ and $0 \le z \le \ell_z = L_z/\mathcal{H}$, and subject to no-slip conditions, $u = w = 0$ on the top and bottom surfaces (but see Section 3), and symmetry conditions on the left and right edges, $u = 0$ and $w_x = 0$. The computational domain is chosen sufficiently larger than the initial shape of Bingham fluid that the precise locations of the upper and right-hand boundaries (*i.e.* ℓ_x and ℓ_z) exert little effect on the flow dynamics.

[1] https://gforge.irsn.fr/gf/project/pelicans/ ; PELICANS is an object-oriented platform developed at the French Institute for Radiological Protection and Nuclear Safety and is distributed under the CeCILL license agreement (http://www.cecill.info/).

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