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Shear-induced particles migration in a Bingham fluid

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ABSTRACT

Shear-induced particle migration in monodisperse and bi-disperse suspension of spherical particles in a Bingham fluid is considered. Previous models of particle migration in Newtonian suspensions are extended to account for the existence of un-yielded regions in the visco-plastic domain when forced to flow in a tube and in a concentric Couette device. In suspension with a monodisperse particle phase, it is shown that particle concentration is continuously augmented in low shear rate regions. The yield boundary is monotonically shifting, affecting the velocity profiles, and the power to maintain the flow is monotonically reduced. When the suspension size distribution is bi-modal, the migration, eventually, results in a separation of the species. Larger particles migrate to the low shear rate zones and the smaller phase is pushed away from there. The velocity profiles, yield boundary and power do not change monotonically, and several stages in this dynamics can be identified.

1. Introduction

Suspensions containing particles dispersed in viscous fluids are common in industrial processes and in materials encountered in daily life. Some of these materials exhibit visco-plastic behavior. Such suspensions flow only if the stress applied surpasses a marginal value known as the yield stress. The visco-plasticity can result from dispersion of particles in a Newtonian fluid, for example as in aqueous suspensions of Kaolin or bentonite (e.g. in [1,2]) or polymers containing a high concentration of fillers, and it may also exists in dispersions of particles in gels that have yield-stress characteristic already, as is for example common in food, pharmaceutical or cosmetic pastes. This manuscript focuses on migration of particles in such suspension.

Processing of such visco-plastic material involves subjecting them to various shear fields. An important parameter of the quality of such processes is the degree of homogeneity of the particle distribution in the continuous matrix of the immerging product. It is well known that particles tend to migrate when a suspension, having a high concentration of inclusions, is subject to shear due to strong interaction of neighboring particles. Such migration can induce inhomogeneity in the particle concentration which can influence the quality of the product. Furthermore, when the inclusions are not of uniform size, it has been predicted that size separation may occur and induce an additional inhomogeneity in the final distribution of the inclusions.

The subject of migration of particles in flowing Newtonian suspensions received considerable attention following the pioneering paper of Leighton and Acrivos [3]. Three basic approaches were proposed to model the migration process. Leighton and Acrivos [3] suggested that two parameters are responsible for the migration process: particle volume concentration and shear intensity. Thus, their model assumed that particles migrate from higher to lower concentration regions and from zones of high shear stress to zones of lower shear stress. The model they suggested contains migration along such gradients which may enhance or compete with each other. Phillips et al. [4] proposed a variation of this model in which the competing parameters are the frequency of particle interaction, involving particle concentration and local shear rate, and the effective viscosity of the suspension, which is dependent on particle concentration as well. Their model is expressed in terms of gradients of these fields. Yet another approach was suggested by Nott and Brady [5], (see also [6]). They treat the dispersed particles as a phase having an effective temperature and an osmotic pressure, and relate the migration effect to rheological properties, in particular to the particle stress in the suspension. It should be noted that the effect of particle induced normal stress was then added to the Phillips et al. [4] model by Krishnan et al. [7] and was expressed in terms of curvature of the streamlines on which the particle interactions occur. It should also be noted that all the models mentioned above contain empirical elements. The desired rigorous account of the microscopic multi-particle interaction (as was calculated for three particle system by Wang et al. [8], which is need for the calculation of particle mean square displacement and diffusion coefficient in concentrated dispersions, is yet to be obtained. Furthermore, rheological characteristics of suspensions

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with non-uniform particle size distribution, and their possible effect on the migration process obtained very limited attention until now.

The migration process in suspensions of particles of various sizes appears to be more complex. Shauly et al. [9] extended the Phillips et al. approach to model migration in poly-dispersed Newtonian suspensions. The effect of particle induced normal stress was added to the Phillips et al. [4] model as was suggested by Krishnan et al. [7], and was expressed in terms of curvature of the streamlines on which the particle interactions occur. For bi-modal distribution of particle sizes they solved for steady state distributions in various viscometric flows and obtained various patterns of particle size separations that agreed with available experimental observations. Such phase separation is expected to be most significant in processing of suspensions containing a particle size distribution. However, the steady state solutions give no information about the dynamics of the migration process and, moreover, the evolution of the separation process.

It is the purpose of this paper to address the process of particle migration and size separation in visco-plastic suspensions. Such dispersions differ from Newtonian ones since there may exist regions in the flow field where the critical yield-stress is not exceeded and where no shear rate exists. Thus, the domains of migration, accumulation, depletion and size separation and segregation are expected to become more complex than those encountered in Newtonian suspensions. We adopt the empirical model of Phillips et al. [4], including the modification proposed by Krishnan et al. [7], and we follow the extension of this model by Shauly et al. [9] to analyze suspensions with poly-dispersed particle size distribution. The dynamic model for migration fluxes is shortly reviewed in Section 2. Sections 3 and 4 contain calculated results for dynamic evolutions in unidirectional viscometric flows, i.e. flow in a circular tube and between rotating cylinders of a Couette device, respectively. A short discussion is given in Section 5.

2. Dimensional model

Consider a suspension of spherical particles of radius a with a volume concentration ϕ. The yield-stress suspension that we address is a Bingham plastic for which the rheology is given by

$$\begin{aligned} &\boldsymbol{\tau} = 2(\mu_P + \tfrac{\tau_Y}{|\mathbf{D}|})\mathbf{D}, && |\boldsymbol{\tau}| \geq \tau_Y, \\ &\mathbf{D} = 0, && |\boldsymbol{\tau}| < \tau_Y. \end{aligned} \tag{1}$$

Here, $\tau_Y$is the yield-stress, $\mathbf{D}$ is the rate of deformation tensor, $\gamma = \sqrt{\frac{1}{2}(\mathbf{D}:\mathbf{D})}$ and μ_P is the plastic effective viscosity. The densities of the fluid and particle phases are equal.

We make several assumptions concerning the rheology of particle migration in this medium. Since there in no data on the dependence of the yield stress, τ_Y, on large changes of the volume concentration of the particles, as a first step, we consider the case where the yield stress does not depend on the particle volume fraction, although in real-life materials this assumption may be violated. We also assume that the migration dynamics of the particles in the yielded Bingham fluid is similar to that encountered in a Newtonian fluid. This strong assumption is yet to be justified. The effective viscosity of the yielded medium is assumed to follow the correlation due to Krieger [10], $\frac{\mu}{\mu_0} = (1 - \frac{\phi}{\phi_m})^{-1.82}$, with μ_0 being the viscosity of the yielded fluid de-voided of particles and with the maximum particle volume concentration of a mono-dispersed suspension being $\phi_{m0} = 0.68$. For a bi-dispersed suspension, the maximum concentration is calculated by the correlation suggested by Probstein et al. [11]

$$\frac{\phi_m}{\phi_{m0}} = \left[1 + \frac{3}{2}|b|^{\frac{3}{2}}\left(\frac{\phi_1}{\phi}\right)^{\frac{3}{2}}\left(\frac{\phi_2}{\phi}\right)\right] \quad \text{with}$$
$$b = (a_1 - a_2)/(a_1 + a_2) \quad a_1 > a_2 \tag{2}$$

that fits well a variety of experimental data with Newtonian suspension. Furthermore, for suspensions with relatively high particle concentration and high effective viscosity, it is henceforth assumed that the viscous flow is inertia-less and that, given the particle concentration distribution and the boundary conditions, the velocity distribution and the shear rate are readily established.

Following Phillips et al. [4] and Krishnan et al. [7], the flux of migrating particles in a suspension of mono-dispersed particles, i.e. particles of the same size, is

$$\mathbf{J} = -k(a^2\phi\nabla(\gamma\phi) + a^2\gamma\phi^2\frac{1}{\mu}\nabla\mu^2 + a^2\gamma\phi^2\nabla\ln R) \tag{3}$$

where k is an O(1) constant. In (3) the three terms are associated with changes of the frequency of interaction, the effective viscosity and the bulk streamline curvature, R, relative to that of the particle.

For a suspension with several discrete particle sizes a_i, Shauly et al. [9] extended the above expression to the following form of the flux of species i

$$\mathbf{J}_i = -k(a_i\sum_j a_j\phi_j\nabla(\gamma\phi_i) + \bar{a}^2\gamma\phi\phi_i\frac{1}{\mu}\nabla\mu^2$$
$$+ \bar{a}a_i\gamma\phi\phi_i\left(\frac{a_i}{\bar{a}}\right)^2\nabla\ln R) \tag{4}$$

Here, $\phi\,\bar{a} = \sum_j a_j\phi_j$ and $\phi = \sum_j \phi_j$.

Since the densities are assumed equal there is no migration due to gravity. The dynamic changes in concentration are obtained from the particle balances

$$\frac{d\phi_i}{dt} = -\nabla\cdot\mathbf{J}_i \quad \text{and} \quad \frac{d\phi}{dt} = -\nabla\cdot\mathbf{J} \tag{5}$$

for species i and for the total concentration (also for the monodisperse case), respectively. This time derivative is a full material derivative which accounts for convective changes in the particle concentration when appropriate, as is the case in the migration described in the next section.

3. Migration in a tube flow

Assume a highly viscous homogenous suspension of spherical particles of initial total concentration ϕ_0 introduced at the inlet of a circular tube under a pressure gradient. In what follows we render variables non-dimensional. Dimensions are scaled by the tube radius, R, the velocity by $Q/\pi R^2$ with Q being the volumetric flux of the homogeneous suspension assumed constant, the shear rate by U/R, the pressure and stresses are scaled by $U\mu_{p0}/R$, and the Bingham number is defined as $Bn = R\tau_y/U\mu_{p0}$. Here μ_{p0} is the initial plastic viscosity of the uniform suspension. For brevity, we use the same characters for the non-dimensional variables.

The non-dimensional rheology has the form

$$\begin{aligned} &\boldsymbol{\tau} = 2(\kappa(r,z) + \tfrac{Bn}{|\mathbf{D}|})\mathbf{D}, && |\boldsymbol{\tau}| \geq Bn, \\ &\mathbf{D} = 0, && |\boldsymbol{\tau}| < Bn. \end{aligned} \tag{6}$$

where $\kappa = \mu_p/\mu_{p0}$, and the evolution of the suspension concentration is measured along the tube in terms of tube radii. The velocity distribution of a homogenous Bingham plastic in a tube, and the relation between the yield stress and the dimension of the un-yielded core at the center of the tube are well known and can be found in textbooks (e.g. Bird et al. [12] pp. 48–50). Due to the strong non-uniform viscosity that evolves along and across