Journal of Non-Newtonian Fluid Mechanics 000 (2016) 1–8

Contents lists available at ScienceDirect

Journal of Non-Newtonian Fluid Mechanics

journal homepage: www.elsevier.com/locate/jnnfm

Particle-induced viscous fingering

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ARTICLE INFO

Article history:
Received 2 March 2016
Revised 27 June 2016
Accepted 27 June 2016
Available online xxx

Keywords:
Viscous fingering
Particle-laden flow
Shear-induced migration
Suspension balance model

ABSTRACT

The Saffman–Taylor fingering instability arises when a less viscous fluid displaces a more viscous one inside porous media, which has been extensively studied for decades. Conversely, the invasion of a more viscous fluid into a less viscous fluid is inherently stable to interfacial instabilities. However, Tang et al. [1] first observed that the addition of particles to a viscous invading fluid can destabilize the fluid-fluid interface, even in the absence of the unstable viscosity ratio. Building on the previous observations, we experimentally characterize the particle-induced fingering patterns in a radial source flow for varying particle volume fractions and gap sizes. The onset of fingering is observed to be highly dependent on the particle volume fraction and also, to a lesser extent, on the channel gap thickness. The key physical mechanism behind this instability is the particle accumulation on the interface that stems from the *shear-induced migration* of particles far upstream of the interface. We model the particle-laden flow as a continuum in the quasi-steady region away from the interface, based on the suspension balance approach, and successfully validate the effects of shear-induced migration on the particle accumulation and subsequent fingering.

1. Introduction

Viscous fingering, one of the seminal problems in fluid mechanics, occurs when a less viscous fluid displaces a more viscous one in porous media [2–4]. Since its discovery, it remains an active area of research [5–7] due to its relevance in geophysical flows and other various phenomena, such as ribbing instability [8], balcony growth [9], dendritic growth [10], and flame propagation [11]. More recently, researchers have expanded the study of viscous fingering by focusing on techniques to control or modify fingering. For instance, the control of the viscous fingering instability was demonstrated by strategically modifying the channel geometry [12] or by modulating the elasticity of the channel walls [5]. Alternatively, viscous fingering can also be modified by changing the fluid phases themselves. For instance, incorporating non-Newtonian fluids, such as shear-thinning [13,14] and Boger [15] fluids, has shown to change the resultant fingering behaviors compared to their Newtonian counterpart.

In this paper, we focus on the viscous fingering instability that is specifically *induced* by the inclusion of non-colloidal particles in the displacing liquid. Given the prevalence of suspension flows in geophysical systems, such as avalanches, as well as in industry that ranges from nanotribology to hydraulic fracturing, the study of interfacial instabilities in the presence of particles represents an exciting and relatively unexplored area of research. Tang et al. [1] first observed an unexpected interfacial instability when a mixture of non-colloidal particles and viscous fluid displaces air in a Hele–Shaw cell. Then, they qualitatively correlated the particle volume fraction, ϕ_0, to the instability growth rate. It is noteworthy that the same flow without particles is inherently stable. More recently, Ramachandran & Leighton [16] observed the analogous fingering instability upon squeezing a particle-oil mixture between two parallel plates.

The mechanism of particle-induced fingering is the direct result of particle accretion on the fluid-fluid interface [1]. As the suspension viscosity is directly related to the local volume fraction, the increase in particle concentration leads to the viscosity gradient susceptible to miscible viscous fingering. The particle enrichment on the free surface has been previously observed in the suspension flow in a tube. For instance, Karnis & Mason measured the rate of particle accumulation on the meniscus in a tube flow [17]. The mechanism of the particle accumulation was later elucidated by Chapman [18] who attributed it to particles' tendency to migrate towards the regions of low shear stress, or *shear-induced migration* [19,20], upstream of the interface. More recently, Ramachandran & Leighton [21] incorporated the effects of gravity on the particle accumulation near meniscus in a tube flow and used the suspension

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http://dx.doi.org/10.1016/j.jnnfm.2016.06.014
0377-0257/

Table 1
Experimental conditions tested; for all experimental runs, the flow rate Q and particle size D are kept constant at 150 mL/min and 125 – 150 μm, respectively. A ϕ_0 range that is denoted as '# - #' increases by an increment of 1%.

h (mm)	ϕ_0 (×100%)
0.6	10–18, 20–30 (increment of 1)
0.7	12–30
1.0	12–15, 17–32
1.2	14, 15, 17, 20–28, 30–35
1.3	8, 11, 14, 17, 20–35
1.4	8, 11, 14, 15, 17, 20–35

balance model [22] to implement the effects of shear-induced migration.

The suspension balance method [22] constitutes one of widely used continuum approaches to model particle-laden flows in the low Reynolds number limit, along with diffusive flux phenomenology [19,20,23,24] and, more recently, frictional suspension rheology that addresses the transition of suspension to granular media [25–28]. The diffusive flux model accounts for the particle migration via semi-empirical relationships between the particle flux and effective suspension viscosity and has been used to successfully model various flow configurations [20,23,24,29,30]. On the other hand, the suspension balance approach describes the particle migration due to the gradient in viscously generated particle stresses [22,31,32]. Distinct from the diffusive flux model, this method captures the rise in anisotropic normal stresses in dense suspensions. Due to its effectiveness in describing viscoelastic characteristics of the mixture in a non-dilute limit, the suspension balance model will be employed in the current analysis.

In the present work, we experimentally quantify the aforementioned particle-induced viscous fingering by injecting suspensions of varying ϕ_0 into a Hele–Shaw cell of varying gap thicknesses. We observe the particle accretion and identify the critical particle volume fraction at which the viscous fingering initiates. Image processing techniques are employed to measure the particle concentration as well as the extent of interfacial deformations. In parallel to experiments, we theoretically confirm the effects of shear-induced migration far upstream of the interface, based on the suspension balance method, in good agreement with the experimental data. The paper is organized as follows: in Section 2.1, we introduce our experimental setup and materials used in the experiments. The experimental analysis and results are summarized in Section 2.2, followed by our theoretical model in Section 3. The paper concludes with the summary and future directions in Section 4.

2. Experiments

2.1. Setup and materials

The Hele–Shaw cell consists of two plexi-glass (acrylic) plates (30.5 × 30.5 × 3.8 cm) that are leveled and separated to a gap thickness, h. The gap separation is controlled by securing shims (McMaster) of different sizes (listed in Table 1) in the four corners of the plates. The mixture is prepared by mixing a PMMS silicone oil (density $\rho_l = 0.96$ g/cm^3 & viscosity $\eta_l = 0.096$ Pa · s, UCT) and neutrally-buoyant polyethylene particles (density $\rho_p = 1.00$ g/cm^3, Cospheric) with diameter, $D = 125 - 150$ μm, to an initial volume fraction, ϕ_0. A syringe pump (New Era Pump Inc., Model NE-1010) is used to inject the mixture into the Hele–Shaw cell at a constant flow rate, $Q = 150$ mL/min. An LED panel (EnvirOasis, 75W, 4200 Lumen) is placed under the Hele-Shaw cell to provide uniform illumination, while a Canon 60D camera (1920 × 1080 pixel images, FOV 64°) records the particle-laden flow from above at 30 frames per second with the spatial resolution of 0.100 ± 0.0075 mm^2/pixel (schematic in Fig. 1(a)). All the experimental parameters tested are summarized in Table 1.

Images collected from the experiments are processed using MATLAB image processing toolbox. Once the images are cropped and smoothed with a median filter, the distance from the injection center to the suspension interface, R_b, (schematic in Fig. 1(b)) is extracted in each image using a built-in edge detection function. The edge detection code utilizes the "Canny" method that computes the local gradient of the image intensity field and identifies the maxima as edges. Unwanted noisy "edges" that do not meet the threshold number of data points are subsequently removed. In addition to computing the evolving shape of the interface, the variation in light intensity is also used to extract the local particle concentration inside the suspension, which will be elaborated in Section 2.2.

2.2. Experimental results

We inject the particle-oil mixture from the center of the Hele–Shaw cell by varying two key parameters: initial particle volume fraction, ϕ_0, and the ratio of gap thickness to particle diameter, h/D. Fig. 1(b) shows a typical image of fingering at $\phi_0 = 0.35$ and $h/D = 10.2$: as particles accumulate on the interface, the fluid-fluid interface deforms with growing particle clusters (see zoomed-in image of Fig. 1(b)). However, fingering does not occur in all conditions, as demonstrated in Fig. 2. At very low volume fractions, the distribution of suspension in the cell is uniform, with no resultant fingering. As ϕ_0 is increased, the particle accumulation on the meniscus becomes visible; finally, beyond some critical value of ϕ_0, the interfacial deformations appear, accompanied by the formation of particle clusters.

Fig. 3(a) illustrates the time-elapsed comparison between typical low concentration ($\phi_0 = 0.14$) and high concentration ($\phi_0 = 0.35$) cases, while all the other parameters (*i.e.* h/D, Q) remain unchanged. At $\phi_0 = 0.14$, the interface remains circular over time, with the uniform distribution of particles throughout. At $\phi_0 = 0.35$, the interfacial deformations and particle cluster formation are first observed around time = 5.8 s. As the interface advances further, the fingering patterns do not significantly change in magnitude, while the particle clusters grow radially. Notably, the spatial distribution of interfacial fingers and particle clusters appears remarkably uniform for all times. In addition, the plot of the instantaneous radius, R_b, as a function of θ in Fig. 3(b) clearly demonstrates the difference between the low and high concentration regimes; R_b at $\phi_0 = 0.35$ strongly varies with θ while that of $\phi_0 = 0.14$ remains relatively uniform, which matches the qualitative observations.

The overall magnitude of interfacial deformation can be computed as a single dimensionless parameter, Λ [16], such that

$$\Lambda = \frac{1}{S}\int_0^S \left(1 - \frac{R_b(s)}{R}\right)^2 ds, \quad (1)$$

where s refers to the curvilinear coordinate defined along the interface, while R is the radius of the best fitted circle of the area occupied by the suspension (notations given in Fig. 1(b)). As Λ characterizes the deviation of the instantaneous interface from a circle, it reduces to zero when $R_b(s) = R$, or the interface forms a perfect circle. Fig. 3(c) shows the plot of Λ versus $R(t)/R_0$ for varying values of ϕ_0. Consistent with our observations, Λ at early times appears to be independent of ϕ_0 for all cases within the margin of error [33]. The effect of ϕ_0 on Λ is evident at later times, as Λ rises much more significantly for larger ϕ_0. However, overall the value of Λ remains in the order of 10^{-4} even for large ϕ_0, suggesting that the interfacial deformations are minimal, compared to