Accepted Manuscript

Long bubbles in tubes filled with viscoplastic fluid

M. Jalaal, N.J. Balmforth

PII: S0377-0257(16)30082-9
DOI: 10.1016/j.jnnfm.2016.06.006
Reference: JNNFM 3800

To appear in: *Journal of Non-Newtonian Fluid Mechanics*

Received date: 8 March 2016
Revised date: 27 May 2016
Accepted date: 8 June 2016

Please cite this article as: M. Jalaal, N.J. Balmforth, Long bubbles in tubes filled with viscoplastic fluid, *Journal of Non-Newtonian Fluid Mechanics* (2016), doi: 10.1016/j.jnnfm.2016.06.006

Long bubbles in tubes filled with viscoplastic fluid

M. Jalaal[a], N.J. Balmforth[b,*]

[a] Department of Mechanical Engineering, University of British Columbia, Vancouver, BC, V6T 1Z4, Canada
[b] Department of Mathematics, University of British Columbia, Vancouver, BC, V6T 1Z2, Canada

Abstract

An analysis is presented of the thin viscoplastic film coating the wall of a slot or tube as a long bubble is displaced down the conduit by ambient fluid flow (the viscoplastic version of a classical viscous problem studied by Bretherton). Lubrication theory is used to analyse the limit of low Capillary number and examine primarily the effect of a yield stress. The predictions are compared with the results of numerical simulations with the open source code Gerris.

1. Introduction

In a seminal paper in interfacial fluid mechanics in 1961, Bretherton [1] considered how a bubble contained in two-dimensional slot or axisymmetrical tube would be displaced down the length of that conduit by an ambient viscous fluid flow. Bretherton used lubrication theory, which applies when the bubble is long in comparison to its width or radius and the fluid films buffering the bubble from the walls are relatively thin. In the limit of small Capillary number (relatively strong surface tension) Bretherton demonstrated how the speed of the bubble was controlled by the thin fluid films, and provided the relation,

$$h_\infty \sim 1.34\, R\, C^{2/3}, \tag{1}$$

where h_∞ is the film thickness over the main bulk of the bubble, R is the slot half-thickness or tube radius and $C = \mu U/\sigma$ is the Capillary number, defined in terms of fluid viscosity μ, bubble speed U and interfacial tension σ. Bretherton's analysis, which is closely connected to the classical Landau-Levich theory for the draw-out of a film from a fluid bath by a plate [2], was subsequently extended to higher Capillary numbers by numerical simulations (*e.g.* [3, 4]) and generalized to a number of generalized Newtonian and viscoelastic fluids [5, 6, 7, 8, 9].

The goal of the current article is to provide a short discussion of the viscoplastic version of Bretherton's problem. In particular, we focus on how a yield stress affects the relation between the residual film thickness and Capillary number. For the task, we use lubrication theory to furnish asymptotic solutions in the limit of small Capillary number. We compare the predictions with numerical simulations using the open source code Gerris. That package cannot properly deal with a yield stress; instead, we regularize the constitutive model and use a bi-viscous law in the computations.

Previous computations and experiments for the propagation of bubbles down tubes filled with flowing viscoplastic fluid have been given by [10, 11, 12, 13]. The existing computations deal with relatively large Capillary number, outside of the regime of validity of Bretherton's lubrication-style theory, and so there is minimal overlap between our results and these previous studies. Also relevant are experiments on the viscoplastic Landau-Levich problem [14, 15] and the work of [16] on the steady motion of viscoplastic plugs down conduits representing idealized airways. A variety of other interfacial flow problems involving bubbles in viscoplastic fluids, of indirect relevance to the present work, are reviewed by [17].

2. Formulation

2.1. Governing equations

Consider a bubble in a slot or tube filled with Herschel-Bulkley fluid. The fluid flows down the conduit transporting the bubble at a speed that differs from the mean fluid speed. The arrangement is assumed symmetric about the midplane or centerline. Conservation of mass and momentum for the velocity field $\boldsymbol{u}$, pressure p and deviatoric stress $\boldsymbol{\tau}$ of an incompressible fluid take the form,

$$\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0, \tag{2}$$

$$\rho\left[\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}\right] = -\boldsymbol{\nabla}p + \boldsymbol{\nabla}\cdot\boldsymbol{\tau}, \tag{3}$$

Here, ρ denotes fluid density and we adopt the Herschel-Bulkley law to relate the deviatoric stress tensor $\boldsymbol{\tau}$ to the deformation rates:

$$\begin{cases} \dot{\boldsymbol{\gamma}} = \boldsymbol{0}, & \tau < \tau_Y, \\ \boldsymbol{\tau} = \left(K\dot{\gamma}^{n-1} + \dfrac{\tau_Y}{\dot{\gamma}}\right)\dot{\boldsymbol{\gamma}}, & \tau \geq \tau_Y, \end{cases} \tag{4}$$

where K is the consistency, n the power-law index, τ_Y is the yield stress, and τ and $\dot{\gamma}$ represent the second-invariants of the stress tensor and $\dot{\boldsymbol{\gamma}} \equiv \boldsymbol{\nabla}\boldsymbol{u} + (\boldsymbol{\nabla}\boldsymbol{u})^T$.

*Corresponding author: *E-mail:* njbmath.ubc.ca, *Telephone:* 604 827 3034