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Dynamic settling of particles in shear flows of shear-thinning fluids☆

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ABSTRACT

Dynamic settling is the phenomenon whereby a relatively dense particle settles through a sheared flow of a non-Newtonian fluid at a speed that depends on the shear rate of the background flow. This means that due to the nonlinear rheology, the settling velocity may vary spatially and temporally as the background shear rate of the suspending fluid varies, an effect which does not occur in Newtonian fluids. In this contribution, the consequences of this dependency are explored for a dilute suspension of particles released uniformly from a source in a sustained and externally-driven flow of shear-thinning fluid. It is shown theoretically that the concentration field does not remain uniform, but evolves downstream, allowing calculation of the runout length, settling times and distribution of the deposited particles. Flows with a velocity maximum are demonstrated to affect the concentration field very strongly as they develop a 'kinematic barrier' over which settling times are considerably lengthened. Flows with bidisperse suspensions are shown to produce deposits that vary non-monotonically in thickness and composition with distance downstream, an effect which is solely due to dynamic settling. Finally flows of viscoplastic fluids which exhibit yielded and unyielded regions may accentuate the role and effects of the kinematic barrier to settling.

1. Introduction

The transport of relatively dense particles in a flow of a shear-thinning or viscoplastic fluid occurs in many natural and industrial settings. Examples include the transport of cuttings by drilling mud [1], proppant emplacement in hydraulic fracturing (e.g. [2], Section 6.2), the transport of coarse material such as sand and gravel in a fine-grained slurry [3,4] or debris flow [5,6] and the handling of two-phase materials in food processing [7]. Key questions for modelling such flows include the distance that particles can be transported before settling out, the time they take to do so, the geometry of the deposit, and the distribution of grain sizes within it.

A small particle suspended in an otherwise quiescent non-Newtonian fluid will typically settle in a laminar regime at a constant speed [8–10], unless the gravitational stress that the dense particle exerts is unable to overcome the yield stress of the suspending fluid [11]. However, if the fluid is not quiescent then the nonlinearity of the rheology means that the background flow affects the settling. In a flow which is sheared on a rather larger scale than the scale of the particle, the background shear rate determines the local viscosity of the fluid, which in turn determines the settling velocity. An extreme case occurs when the fluid has a yield stress, in which case small suspended particles may be supported indefinitely when the fluid is static, but then settle when the fluid is set in motion by forces that exceed the yield stress.

This dependence of settling velocity on the background flow is sometimes referred to as 'dynamic settling' and has been known for many years in the oil and process engineering communities [2,12–15]. Because it allows particles to be carried long distances in relatively low-shear flows, it can be a desirable effect in, for example, the transport of drill cuttings; however, it is among the factors implicated in undesirable phenomena such as barite sag [16]. (Confusingly, it may act in conjunction with Boycott settling, also referred to in the literature as 'dynamic settling'.) Under the name of 'competence variation' [5], it has also been proposed as a mechanism for inverse grading in deposits from muddy flows.

Despite these applications, dynamic settling has hitherto received relatively little attention from fluid dynamicists. Notable experimental contributions have included those of Merkak et al. [17,18], who studied experimentally the flow and sedimentation of small particles suspended in a viscoplastic gel, and that of Ovarlez et al. [19] has examined experimentally the shear-induced sedimentation of relatively small particles in yield stress fluids using MRI to determine the evolving concentration within a sheared Couette device. The most substantial modelling contribution has been that of De Angelis and Mancini [20] (and see also the review by [4]), who used an empirical settling velocity correlation to

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calculate the trajectories of particles settling within a viscoplastic pipe flow and feeding a mobile dense layer at the bottom of the pipe, under which a static deposit grew.

In this contribution, we analyse a simple model of dynamic settling in horizontal shear flows of a shear-thinning fluid, and investigate the fate of particles as they are transported downstream and settle out of suspension. This model reveals that dynamic settling may have a number of interesting consequences, including strong effects on transport distances, the development of statically unstable particle concentration gradients within the flow, and the formation of deposits that neither thin nor become finer monotonically with distance downstream from the source. The latter two effects cannot occur with particles settling from a flowing Newtonian fluid, for which the settling velocity is constant and increases monotonically with particle size. Instead the development from a uniform source of statically unstable concentration gradients and spatially varying deposits is due to dynamic settling. The vital coupling in shear flows of shear-thinning fluids that leads to these phenomena is that the settling velocity is reduced in regions of low shear rates and thus these zones can act as 'kinematic barriers' to settling particles.

The situation is more complicated when the suspending fluid possesses a yield stress, because relatively heavy particles could be fully supported within unyielded regions and even if they are of a sufficient submerged weight to overcome the yield stress, their sedimentation is affected by the yield stress [11]. In this study, we examine theoretically how a yield stress, and associated unyielded regions within a flow with spatially varying shear rates, influence dynamic settling. We focus on particles that are not arrested within the unyielded regions (unlike [20]) and we examine how the yield-stress effects complement the kinematic barrier due to the shear-thinning properties of the fluid.

We formulate a model for the flow in Section 2, basing our exposition on power-law fluids. We show in the appendix that an equivalent analysis can be carried out for fully developed flows of any generalised Newtonian fluid, but for the purposes of discussing the interplay of dynamical processes in these flows, we employ the simple power-law rheology (and later when analysing viscoplastic flows, the Herschel–Bulkley rheology). We tackle theoretically three related problems, which illustrate the consequences of dynamic settling. First we analyse settling within a horizontal free-surface flow driven by a constant pressure gradient (Section 3). We show how a non-uniform distribution of concentration arises due to dynamic settling effects. We then analyse the suspension within a two-dimensional channel, also driven by a constant pressure gradient (Section 4). The imposition of a no-slip condition at the upper surface retards the fluid motion and introduces an interior velocity maximum. We show that the maximum distance propagated by the particles within the channel flow is always less than in a free-surface flow of the same depth, driven by the same pressure gradient, but that the time taken for full settling to occur is always increased. However, somewhat counter-intuitively, for strongly shear-thinning fluids, the median of the depositional flux can be further from source in the channel flow than in the free-surface flow. We analyse a dilute bidisperse suspension (Section 5) and show that dynamic settling alone can lead to compositional variations within the deposit. Finally in Section 6 we demonstrate the effects of a yield stress on settling through a horizontal channel flow, focusing on the role of the unyielded plug at the centre of the channel.

2. Formulation

We study the sedimentation of dilute suspensions of relatively dense particles within a shear flow of a non-Newtonian fluid, the motion of which is driven by an imposed horizontal pressure gradient. The particles are transported by the flowing interstitial fluid, but due to their excess density settle under the action of gravity and form a deposit on the underlying boundary.

We analyse the motion of the fluid and suspension in two spatial dimensions, with the coordinate axes aligned such that the x axis is horizontal and streamwise, while the z axis is vertical; unit vectors aligned with the x and z axes are denoted by $\hat{\mathbf{x}}$ and $\hat{\mathbf{z}}$, respectively. The fluid motion is steady and fully developed so that the velocity field is given by $\mathbf{u} = u(z)\hat{\mathbf{x}}$. It is driven by a sustained pressure gradient, $-G\hat{\mathbf{x}}$. The deviatoric shear stress is denoted by τ_{xz} and momentum balance leads to

$$\frac{\partial \tau_{xz}}{\partial z} = -G. \tag{1}$$

The concentration of particles suspended in the fluid is denoted by $C(x, z, t)$ and the equation governing its evolution is

$$\frac{\partial C}{\partial t} + \nabla \cdot (\mathbf{u}_p C) = 0, \tag{2}$$

where the velocity field with which the concentration is advected is denoted $\mathbf{u}_p$. The diffusivity of the solid phase has been neglected because the particles are assumed to be sufficiently large that they are not affected by molecular fluctuations within the fluid. The suspension is dilute so that there are negligible interactions between the individual particles, and the inertia of the particles is also assumed negligible, so that the drag and gravitational forces that act upon them are in balance. Thus we write the instantaneous relationship between the velocity of an individual particle, the velocity of the fluid and gravitational settling

$$\mathbf{u}_p = u\hat{\mathbf{x}} - w_s\hat{\mathbf{z}}. \tag{3}$$

In contrast to settling through fluid of Newtonian rheology, the settling velocity of the particles depends upon the motion of the interstitial fluid and the consequences of this dependence will be explored below. The particles settle out of the flow to the underlying boundary and build up a deposit of thickness $\eta(x, t)$. Its growth is determined by the settling flux at the boundary and is given by

$$(1 - \phi_b)\frac{\partial \eta}{\partial t} = w_s C(x, \eta, t), \tag{4}$$

where ϕ_b is the volume fraction of particles within the deposit.

In this study we calculate the unsteady development of the suspension and the deposit due to a sustained source of particles imposed at $x = 0$ and initiated at $t = 0$. We thus impose that $C(0, z, t) = C_0$ and that initially the flow is otherwise free of particles, $C(x, z, 0) = 0$. The deposit is also initially of vanishing thickness ($\eta(x, 0) = 0$). Since the flows are dilute, the growth of this deposit does not significantly alter the geometry of the boundary unless the motion is sustained for a long duration, and consequently it does not feed back upon the motion of the fluid phase. Expressed dimensionally this criterion requires that the deposit depth is much less than the flow depth ($\eta \ll h$), which in turn demands that the duration of the flow must be much less than $h(1 - \phi_b)/(w_s C)$.

To progress we adopt a particular rheology; the analysis could be performed rather generally (see Appendix A), but for the simplest exposition of the ideas, we focus on a power-law rheology, which encompasses the key feature of shear thinning—and it is this property that plays a vital role in what follows. We therefore assume that the interstitial fluid is of power-law rheology with flow index n and consistency K_n, which is extended to include a yield stress in Section 6. The fully developed flow is then governed by

$$\frac{\partial}{\partial z}\left(K_n \left|\frac{\partial u}{\partial z}\right|^{n-1} \frac{\partial u}{\partial z}\right) = -G. \tag{5}$$