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Particle settling in yield stress fluids: Limiting time, distance and applications

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ABSTRACT

We examine the problem of a single heavier solid particle settling in a yield stress fluid that behaves as a classical Bingham plastic. The flow configuration we are interested in is the transient dynamics from a particle settling in a Newtonian fluid to a Bingham plastic. Depending on the magnitude of the yield stress (or dimensionlessly the Bingham number), the particle and the surrounding fluid may return to rest in a finite time or reach another steady but lower settling velocity. At the analytical level, we write the total kinetic energy decay of the system. We evidence the existence of a critical Bingham number beyond which motion is suppressed and derive upper bounds for the finite stopping time as well as the maximum path length. These estimates can be obtained in 2D only while the extension to 3D remains an open question. At the numerical level, we design a robust and efficient Lagrange multiplier based algorithm that enables us to compute actual finite time decay. The algorithm combines an Augmented Lagrangian outer loop to treat the exact Bingham law to a Distributed Lagrange Multiplier/Fictitious Domain inner loop to account for freely-moving particles. We show that the ability to compute the balance between net weight (weight plus buoyancy) and yield stress resistance is the key point. The algorithm is implemented together with a Finite Volume/Staggered Grid algorithm in the numerical platform PeliGRIFF. We investigate 2D configurations with the following particle shape: (i) a circular disc and (ii) a 2:1 rectangle.

1. Introduction

The settling of a particle in a yield stress fluid is the result of the balance between weight (the driving force, downwards oriented) and buoyancy (static pressure), dynamic pressure, viscous and yield stress resistance (all 4 upwards oriented). For a fixed net weight (weight plus buoyancy, i.e., weight computed using the density difference), the increase of the yield stress results in a lower particle settling velocity. Therefore, it is rather intuitive that there exists a critical yield stress such that the net weight cannot overcome the yield stress resistance and the motion is suppressed. In a dimensionless context, this critical yield stress translates into a critical Bingham number. This property has been evidenced experimentally by many authors. The ability of a heavier particle or many heavier particles (suspension) to settle or not settle in a yield stress fluid can be seen in some processes or flows either as a constraint or as an interesting property to exploit. For instance, in industrial applications, the suspending yield stress fluid can be designed in such a way that settling is suppressed (cuttings removal in drilling operations, slurry transport in mining, etc). In case of a poly-disperse suspension, the yield stress property of the suspending fluid can be exploited to screen particles of different size and/or different density.

The general problem of estimating the settling velocity of a solid body of arbitrary shape in a yield stress fluid as well as the critical Bingham number beyond which the particle is motionless and the whole fluid is unyielded is both of practical and fundamental interests. Experimentally, the flow configuration is pretty elementary (although thixotropy might be a problem in supplying reproducible results and most viscoplastic materials exhibit a certain level of reversible elasticity below the yield stress). In a tank filled with a given yield stress material of reasonably well controlled yield stress, releasing objects of various size and density provides a nice setting to measure both the settling velocity and the critical Bingham number beyond which motion is suppressed [1–4]. Understanding the behavior of a single object (generally a

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sphere) settling in a yield stress fluid represents the first step to investigating more complex phenomena in suspensions of particles in a yield stress fluid as e.g. shear-induced sedimentation [5].

At the analytical level, writing the whole problem in a variational way enables one to define the critical Bingham number as the solution of an optimization problem, i.e., the supremum of the rate of doing work of the particle net weight over the plastic dissipation in the fluid. To determine a theoretical value of the critical Bingham number, an admissible velocity field is required in the variational principle. This can be achieved for simple solid body shapes only, essentially a circular disc in 2D and a sphere in 3D. In particular, in the case of a disc, this critical Bingham number has been estimated to be 0.0658. Bounding the kinetic energy decay, an upper bound of the time required for the particle and fluid system to return to rest can be established. This bound can actually be derived in 2D only as in 3D the plastic dissipation term cannot be bounded from below in the right functional space. We will derive these equations in Section 3.

At the numerical level, the problem of a single particle settling in a yield stress fluid has received a significant attention in its reverse configuration, i.e., the flow past a fixed obstacle. Mostly spherical in 3D [6–8] and circular in 2D [9–13] obstacles have been considered, although these simulations could have easily been extended to any kind of shape. Keeping the particle fixed and imposing an external flow simplifies the computations as it removes one difficulty, the need to handle a freely-moving particle (that actually exists regardless of the rheological nature of the surrounding fluid). However, the second difficulty associated to the numerical treatment of the yield stress constitutive equation is still present. As is now considered standard in the literature to compute the flow of a yield stress material in a fixed domain, two different types of solution methods are employed. The former relies on the regularisation of the gradient discontinuity and undetermined value of the stress below the yield stress in the constitutive law. In essence, regularisation methods involve using a smooth stress-strain rate purely viscous relation with an artificially high viscosity below the yield stress. The latter recasts the problem into the minimization of a functional and enables one to keep the true constitutive law (for more details the interested reader is referred to [14]). Although the flow past an obstacle is an appropriate flow configuration to estimate the critical Bingham number beyond which flow does not exist, it does not permit to address the question of return to rest in finite time as generally a velocity boundary condition is imposed in the solid body (or on the far field velocity) instead of a body force. In the special case of a disc in 2D and a sphere in 3D assumed to have a simple translational motion along the direction of gravity, the particle translational velocity can be introduced as an unknown and computed via a simplified force balance. To solve the full force balance on a solid body of arbitrary shape, one needs to consider the freely-moving particle case. Assorted numerical methods have been suggested in the literature to model the free motion of a solid particle immersed in a fluid [15–20]. One first step towards the accurate simulation of freely-moving solid bodies in a yield stress fluid consists in simply coupling a numerical technique for the viscoplastic nature of the surrounding fluid to another technique for the free motion of the particles. This has been attempted in [15,21]. Although these two contributions to the literature indeed supplied valuable simulation results for the flow of a single or two interacting spheres in a yield stress fluid, the adopted numerical methods were neither capable of simulating a situation where a particle remains motionless over an infinite time as a result of the yield stress resistance being larger than the net weight, nor the finite time decay of the flow (transition from a particle settling at a certain non-zero velocity to rest in finite time as a result of an increase of the yield stress beyond the limit of flow). In fact, the former [21] did not use the adequate time algorithm and the latter [15] anyway used a regularized constitutive law, that prevents from getting finite time decay even in a fixed domain. Not only considering the actual yield stress constitutive equation is required, but also solving implicitly the equations is crucial to properly satisfy the force balance on the solid particle. An adequate time algorithm is hence the key ingredient.

An outline of our paper is as follows. In Section 2, we formulate the dimensionless problem to be studied. The variational formulation is introduced in Section 3 and we outline the general properties of the solutions, e.g. symmetry, reversibility, uniqueness. We explore the relation between mobility and resistance problems, for a Bingham fluid, and derive general results on the static stability limit (or load limit). In Section 4, we elaborate on the way we construct an approprite semi-implicit solution method using a combination of two Lagrange multiplier based numerical techniques. Also in Section 4, the general analytical results are applied to the problem of a circular disc and a 2:1 rectangle settling in a 2D planar domain, considering in particular the limit of zero flow at sufficiently large Bingham number. The paper ends with a brief discussion.

2. Problem statement

We consider the motion of a rigid particle P within a viscoplastic fluid in large but finite domain Ω. The fluid satisfies the following momentum and mass conservation equations:

$$\mathrm{div}\ \hat{\boldsymbol{T}} - \nabla \hat{p} + \hat{\rho}_f \hat{\boldsymbol{g}} = \hat{\rho}_f \frac{D}{D\hat{t}} \hat{\boldsymbol{u}}, \quad \text{in } \Omega \setminus \overline{P}, \tag{1a}$$

$$\mathrm{div}\ \hat{\boldsymbol{u}} = 0 \quad \text{in } \Omega \setminus \overline{P}, \tag{1b}$$

$$\hat{\boldsymbol{\sigma}} = -\hat{p}\boldsymbol{\delta} + \hat{\boldsymbol{T}}, \tag{1c}$$

where $\hat{\boldsymbol{u}}$ denotes the fluid velocity in the fluid domain $\Omega \setminus \overline{P}$, $\hat{p}$ is the pressure, $\hat{\boldsymbol{\sigma}}$ is the total stress, $\hat{\boldsymbol{T}}$ is the deviatoric stress tensor, $\hat{\boldsymbol{g}}$ is the gravitational acceleration, of magnitude $\hat{g}$, and $\hat{\rho}_f$ is the fluid density. Throughout the paper we shall adopt the convention of denoting dimensional quantities and variables with the "hat" symbol, i.e., $\hat{\cdot}$.

Our main interest is in the interaction of the net weight with the yield stress. We therefore model the fluid using the Bingham model; see [22–24].

$$\begin{cases} \hat{\boldsymbol{T}} = \left(\hat{\mu} + \frac{\hat{\tau}_y}{\hat{\dot{\gamma}}(\hat{\boldsymbol{u}})}\right)\hat{\dot{\boldsymbol{\gamma}}}(\hat{\boldsymbol{u}}), & \text{if } \hat{T} > \hat{\tau}_y \\ \hat{\dot{\boldsymbol{\gamma}}}(\hat{\boldsymbol{u}}) = 0 & \text{if } \hat{T} \leq \hat{\tau}_y \end{cases}, \tag{2a}$$

where $\hat{\tau}_y$ and $\hat{\mu}$ are the yield stress and plastic viscosity of the fluid, respectively. The tensor $\hat{\dot{\boldsymbol{\gamma}}}(\hat{\boldsymbol{u}})$ is the rate of strain tensor associated with the velocity field $\hat{\boldsymbol{u}}$, defined component wise as

$$\hat{\dot{\gamma}}_{ij}(\hat{\boldsymbol{u}}) := \frac{\partial \hat{u}_i}{\partial \hat{x}_j} + \frac{\partial \hat{u}_j}{\partial \hat{x}_i}, \quad \hat{\boldsymbol{u}} = (\hat{u}, \hat{v}, \hat{w}) = (\hat{u}_1, \hat{u}_2, \hat{u}_3),$$
$$\hat{\boldsymbol{x}} = (\hat{x}, \hat{y}, \hat{z}) = (\hat{x}_1, \hat{x}_2, \hat{x}_3). \tag{2b}$$

$\hat{\dot{\gamma}}$ and $\hat{T}$ are (Euclidian) norms of $\hat{\dot{\boldsymbol{\gamma}}}$ and $\hat{\boldsymbol{T}}$, defined as

$$\hat{\dot{\gamma}}(\hat{\boldsymbol{u}}) = \|\hat{\dot{\boldsymbol{\gamma}}}\| = \sqrt{\frac{1}{2}\sum_{ij} \hat{\dot{\gamma}}_{ij}^2(\hat{\boldsymbol{u}})} \quad \text{and} \quad \hat{T} = \|\hat{\boldsymbol{T}}\| = \sqrt{\frac{1}{2}\sum_{ij} \hat{T}_{ij}^2}. \tag{2c}$$

The main forces acting on P are localised at the particle, due to gravity. We therefore consider that the fluid is quiescent in the far-field, in all directions, and locate the outer boundary Γ in the quiescent zone. For yield stress fluids the precise location is not as important as for purely viscous fluids, because the motion of