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Augmented Lagrangian simulations study of yield-stress fluid flows in expansion-contraction and comparisons with physical experiments

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ABSTRACT

We present numerical simulations of viscoplastic flows in expansion-contraction geometry and compare them with physical experiments of [Chevalier et al. Europhys. Lett. 102, 48002 (2013)] and [Luu et al. Phys. Rev. E 91, 013013 (2015)]. Numerical resolution is done with Augmented Lagrangian (following the Glowinski and coworkers' approach) and Finite-Differences (for the space discretization) methods. We show that good agreement is obtained between the numerical results and the physical experiments. In particular, we retrieve the slip line effect of Luu et al. and give numerical evidence of non-monotone shear effect in the boundary layer between the two unyielded regions in the cavity region. We also give some more detailed measures of the size of the plug and dead zones.

1. Introduction

In the present article, we study the ability of Augmented Lagrangian methods to simulate two dimensional flows of viscoplastic materials in rectangular expansion-contraction geometries. We are specifically interested in the numerical simulation of recent physical experiments of Chevalier et al. [6] and Luu et al. [19]. We provide a detailed analysis of the velocity profiles and unyielded zones.

Even if the fluids used in the above experiments are described by a Herschel–Bulkley law, we restrict ourselves to a Bingham constitutive law since, as mentioned in the PhD thesis of Chevalier, it still allows to have good insight of these viscoplastic flows. This will be confirmed in the present article. Moreover for such experiments, flows are studied when reaching a stationary state.

Precisely, we thus want to solve the following 2D stationary, so called, Stokes–Bingham problem:

$$\begin{cases} -\nabla.\tau + \nabla p = 0 \\ \nabla.\mathbf{u} = 0, \end{cases} \tag{1}$$

where τ is given by the Bingham constitutive law:

$$\begin{cases} \tau = 2\eta D(\mathbf{u}) + \tau_y \dfrac{D(\mathbf{u})}{|D(\mathbf{u})|} \Leftrightarrow D(\mathbf{u}) \neq 0 \\ |\tau| \leqslant \tau_y \Leftrightarrow D(\mathbf{u}) = 0. \end{cases} \tag{2}$$

The viscosity of the viscoplastic material is denoted by η and the yield stress by τ_y. We denote by $D(\mathbf{u})$ the rate of deformation tensor: $D(\mathbf{u}) = (\nabla\mathbf{u} + \nabla\mathbf{u}^t)/2,\ \mathbf{u} = (u, v)$ and by p the pressure. We also use the following convention: for a tensor τ, we use the norm $|\tau|^2 = \frac{1}{2}\sum_{ij}\tau_{ij}^2$. The stress of the material is below the τ_y threshold when the material is rigid ($D(\mathbf{u}) = 0$, also called the unyielded state). On the contrary, the material is deformed with a linear law for any stress above τ_y. This kind of viscoplastic formalism originated independently from the works of Schwedoff [31] and Bingham [3], and was then extended to the 3D tensorial form by Prager [17].

The numerical simulation of Bingham flows generated a wide variety of methods to deal with the main difficulty of such problem, namely the fact that the constitutive law is multivalued when the stress is below τ_y. For an extensive review, we refer to the book of Glowinski and Wachs [15]. In brief, one can distinguish two families of approaches: on the one hand, regularization approaches which make the Bingham law univalued and allow to solve (1) and (2) in the strong form, using classical methods as for the incompressible Stokes equation. Of note, even if sometimes very interesting from the theoretical PDE point of view, regularization approaches may lead from the computational viewpoint to

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wrong computation of the yield surfaces associated to the exact Bingham model, see [12] for a review: an example of such problem is given by Burgos et al. [4] where a simulated yield surface has the inverse convexity of the true expected analytic yield surface.

On the other hand, one can use variational approaches where (1) and (2) is reformulated as a variational inequality which allows to solve more precisely for the rigid zones. They can be traced to the works of Il'iushin [18], Prager [24], Mosolov and Miasnikov [20] and Duvaut and Lions [10]. Efficient numerical techniques where designed following the works of Glowinski, Lions and Trémolières [10] and coworkers, including the so called Augmented Lagrangian (AL) methods which are used in the present paper and will be described in the following section. We refer again to [15] and references therein for numerous applications of AL methods in the simulation of viscoplastic flows.

Of course, since it is widely used in practical applications, the expansion-contraction geometry has been studied in many previous works. Let us mention the work of de Souza et al. [7] which seems to be the first work close to the present study: they present experiments with Carbopol and compare with numerical solutions obtained with a regularization method. In addition, similar configurations are simulated in [28] and [29] using an AL method on unstructured meshes. Their code is implemented with the excellent Rheolef library [30] of Saramito and coworkers (see e.g. [26] for the flow around a cylinder). An impressive range of Bingham numbers, aspect ratios of the geometry and shapes of the cavity (rectangular, sinusoidal wave, triangular, semi-fractal) are presented. But they did not describe in depth the velocity profiles in conjunction with the plug zone, along the lines of the physical experiments of Coussot's and Chambon's groups [6,19].

The characteristics of the present paper are the following.

- As said previously we use an AL approach and we adopt a Finite-Difference approach (on Cartesian meshes) for the discretization in space. This is in the spirit of Wachs and coworkers (see [32] or, for a longer description, [15]), as well as E. Muravleva, A. Muravleva, Olshanskii and coworkers (see e.g. [21] and [22]) but our implementation differs on the resolution of the induced generalized Stokes problem which is here also tackled with another AL approach (to fulfill the incompressibility condition). See Section 2.2. In addition, we make a finely tuned use of parallel linear system solvers which helps in using very fine (isotropic) Cartesian meshes, not that often published in the simulation of viscoplastic flows considered here.
- Code results are scrutinized in terms of accuracy of the localization of the plastic zone and computational times, given the fact that we impose a really small residue ($\sim 10^{-12}$) in the AL loop : such information are rarely given in the associated literature and can serve for future comparisons.
- As a validation/application of the code, we retrieve in Section 3 the results of the *frustrated* regime studied in [6] and additionally show the evolution of the yielded boundary layer width as a function of the Bingham number. We also retrieve the existence of a so-called *slip line* and the Poiseuille-like behavior above this slip line shown in [19] (see Section 4). Of note, we also give the horizontal length of the dead zone at the corner of the cavity as a function of the Bingham number (Section 2.3).

2. Expansion-contraction channel simulations

2.1. Description of the problem

The geometry and notations of the expansion-contraction problem are illustrated in Fig. 1, where only the upper half is shown. In the following, we will use either (1) and (2) or their dimensionless form (by denoting dimensionless variables with a *tilde* symbol) which reads:

$$\begin{cases} -\tilde{\nabla}.\tilde{\tau} + \tilde{\nabla}\tilde{p} = 0 \\ \qquad \tilde{\nabla}.\tilde{\mathbf{u}} = 0, \end{cases} \tag{3}$$

with

$$\begin{cases} \tilde{\tau} = 2\tilde{D}(\tilde{\mathbf{u}}) + B\dfrac{\tilde{D}(\tilde{\mathbf{u}})}{|\tilde{D}(\tilde{\mathbf{u}})|} \Leftrightarrow \tilde{D}(\tilde{\mathbf{u}}) \neq 0 \\ |\tilde{\tau}| \leqslant B \Leftrightarrow \tilde{D}(\tilde{\mathbf{u}}) = 0. \end{cases} \tag{4}$$

In this dimensionless Stokes–Bingham model, there is a unique dimensionless number $B = \frac{\tau_y D}{\eta \bar{U}}$, called the Bingham number, where D is the small channel half-width (see Fig. 1) and $\bar{U}$ is the mean flow velocity in the x-direction at the entrance (see (5)). Indeed, the dimensionless model is obtained from (1) and (2) by scaling the lengths with D, the velocities with $\bar{U}$ and stresses with $\frac{\eta\bar{U}}{D}$. In dimensional variables, we have

$$\bar{U} = \frac{1}{D}\int_0^D u(0, y)\mathrm{d}y. \tag{5}$$

We consider the two following aspect ratios:

$$h = \frac{D+H}{D} \qquad \text{and} \qquad \delta = \frac{D}{L}. \tag{6}$$

In the inlet and outlet, we set the flow equal to the Poiseuille flow (with a unit net flux) in the infinitely long channel. At the lateral wall, the velocity is set equal to 0. See Appendix A.2 for details. Of note, in Section 3, we will present the results in the dimensionless form, but we will use the dimensional form in Section 4 to compare more easily with the results of [19].

To sum up, in dimensionless variables, the free parameters are h, δ and the Bingham number B.

2.2. Salient features of the numerical results

As said in the introduction, we implemented an Augmented Lagrangian method as in the seminal work of Glowinski and coworkers [8,14]. The discretization in space is done with Finite-Differences on rectangular grids. As such, present work is complementary to [28,29] since it allows to compare the results between structured and non-structured grids discretizations. For completeness and reproducibility of the paper, we give in Appendix A the algorithms we implemented with Fortran 90 and MPI.

The first key point is that the simulations presented in the paper are much more converged in terms of the AL iterations than many of the associated simulations previously published. For instance, instead of enforcing a convergence of 10^{-6} for the Bingham AL loop's convergence criterion, we used $6 \cdot 10^{-12}$ (and also validated the code up to machine precision 10^{-15}). The second important point is that the linear systems which need to be solved are handled by the MUMPS library [1,2]. This massively parallel solver allows us to use very fine meshes up to $7.8 \cdot 10^6$ points and to obtain computational times shorter than 2 days on 16 cores.

Fig. 2 shows typical computed velocity, pressure and $|\tilde{d}|$ (which approximates $|\tilde{D}(\tilde{\mathbf{u}})|$ as shown in Algorithm 1, cf. Appendix A.1) fields, for $\delta = 0.5$, $h = 2$ and $B = 5$. We directly remark that velocity, pressure and deformation are symmetric with respect to both middle axis in the $\tilde{x}$ and $\tilde{y}$ directions (and it is the same for the stress tensors). Hence, often in the sequel, we only show the upper-left quarter of the domain. Further, as often done in the literature, we cover the plastic zones in the stress fields with a black patch since there's no consistent notion of pressure or stress in the rigid zone, for the Bingham model (1) and (2).