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On the effect of eccentricity and presence of multiphase on flow instability of fully developed flow through an annulus

Satish Kumar Dewangan*, Shobha Lata Sinha

Mechanical Engineering Department, National Institute of Technology, Raipur, CG 492010 India

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ABSTRACT

Flow instability or transition from laminar to turbulent for the various fluids is a subject of research for the fluid dynamics. Various approaches have been devised to look into the various aspects of the transition from laminar to turbulent flow. The present investigation deals with the application of Energy gradient method to study the effect of the eccentricity and presence of the secondary solid phases in the flow instability for the fully developed annular flow. For the annulus of different radius ratio, the different eccentricity ratio has been considered for the Newtonian fluid. Also for the concentric case, the effect of the solid secondary phase has been studied with the possibility of the solid deposition at the bottom of the annulus. These cases have practical relevance with the annular flow in the well bore in case of horizontal drilling. The derivations have been carried out for all these cases for flow instability analysis.

1. Introduction

The topic of the investigation of the transition from laminar to turbulence is inspired from the developments needed in the many industrial situations e.g. oil and gas drilling industries, the heat exchangers, pulp & paper industries, mining industry and chemical process industries etc. The present study is an investigation of the transition from laminar to turbulence of a fully developed flow for Newtonian fluid through annulus for different eccentricity ratio. Along with this, effect of multiphase has also been explored in the fully developed annular flow with Newtonian fluid as base fluid. Operating in the transitional flow regime may prove to be advantageous in multiple numbers of practical flow applications because the large velocity fluctuations available (due to turbulence) can be used to stir up sedimented particles [48].

It is very much important part of the modeling to know the type of flow in the borehole annulus. Procedure of modeling and calculation of the flow parameters are based on the type of flow. As far as bore hole-cleaning is concerned, it is safe to operate the drilling fluid flow in the completely turbulent regimes of the flow. The liquid – solid slurry, flowing through the well bore annulus, is formed by the combination of the drilling fluid (water with some additives) and the cuttings generated. The drilling fluid is usually of non-Newtonian behavior and it acts as carrier media for the cutting generated by the drill bits while drilling operation. Such slurry flow should be in turbulent condition to serve the purpose of hole-cleaning. In the present investigation, water is considered as the carrier fluid, instead of some kind of the non-Newtonian fluid for the purpose of simplification in the derivations of the relevant relationships – without compromising the basic physics. Also in the case of the horizontal well bore, the drill pipe tends to sag in the well bore and thus forms eccentric geometrical situation of the annular flow geometry. Hence, it is mandatory to know the transition from the laminar flow especially for the flow though horizontal well bore where combination of eccentricity and slurry flow are important considerations.

Already many experimental, theoretical and computational researches have been conducted on the topic of studying the transition from laminar to turbulent flow [7,43]. As a result, multiple number of transition criteria for Newtonian and non-Newtonian fluids have been developed as well as mechanism have been explored towards achieving generalized understanding of transition.

Flow instability criteria, location of its nucleation and mechanism depends both on the geometry of the flow as well as upon the properties such as rheology etc. of the fluid, apart from other factors. Rheology of the drilling fluid is the major fluid property that affects the transition behavior of the drilling fluid. Rheology of the drilling slurry matches with the various rheological models for the time dependent and time independent fluids. For all the fluids, the nature of the flow is governed by the relative importance of the viscous and inertial forces. This balance is characterized by Reynolds number *(Re)*, for the case of Newtonian fluid. The gen-

* Corresponding author: Fax: 07712254600.
E-mail addresses: skdewangan.amech@nitrr.ac.in, satish_dewangan@yahoo.com (S.K. Dewangan), slsinha.mech@nitrr.ac.in (S.L. Sinha).

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List of Symbols

E	Total energy for incompressible flows
z	Coordinate in the direction of gravitational field
s	Stream wise directional coordinate
n	Normal direction coordinate, flow behavior index
r	Radial coordinate
ϕ	Azimuthal coordinate
ρ	Fluid density
g	Gravitational acceleration
V	Bulk flow velocity
V_{av}	Average velocity of the annulus
v_z	Axial velocity distribution for the concentric annulus
v_{z-e}	Axial velocity distribution for the eccentric annulus considering the effect of the pressure gradient variation
v_{z-ecc}	Axial velocity distribution for the eccentric annulus considering the effect of the pressure gradient variation and fluid bulging effect, both
v_{max}	$-\frac{1}{4\mu}\ \frac{\partial p}{\partial z}\ R^2$
p	Hydrodynamic pressure
K	Dimensionless stability parameter
K_{max} and K_{min}	Maximum and minimum value of K
K_c	Critical value of K
μ	Dynamic viscosity of the fluid
C_i	Axial pressure gradient correction factor
R_i	Inner wall radius for the annulus
R, R_o	Outer wall radius for the annulus
ε	Eccentricity ratio
e	Eccentricity
$d_i d_o$	Inner and outer wall diameter for the annulus
D_H	Hydraulic mean diameter of the annulus; $2(R_o - R_i)$
k	Radius ratio
b	A Constant; $\frac{(1-k^2)}{\ln(1/k)}$
Q	Volume flow rate;
Re	Bulk Reynolds number for the annulus;
Re_o	Reynolds number of the virtual pipe flow; $\frac{\rho\ (v_{max}/2)\ 2R}{\mu}$
$(\frac{\partial p}{\partial z})_c$	Axial pressure gradient in concentric annulus
μ_s	shear viscosity of the suspending fluid
μ_{rel}	Relative viscosity for the simple shear flows [the ratio of the shear viscosity of the suspension (μ) to the shear viscosity of the suspending fluid(μ_s)]
ϕ	Slurry concentration
Suffix '*con*$'	Concentric annulus
Suffix '*ecc*$'	Eccentric annulus

erally accepted value for Reynolds number Re is 2100, after which the stable laminar flow does not exist any longer. For time independent fluid, critical Re value depends upon the type and degree of non-Newtonian behavior. The property of the slurry i.e. mixture of the liquid and solid depends upon the various factors such as individual physical properties of the solvent (carrier phase) liquid & solute (secondary solid phase), particle size & shape, degree of the solubility / homogeneity of the solution formed etc. Thus slurries can be classified as homogeneous and heterogeneous. For homogeneous slurry, the slurry solution behaviors can be defined approximately by non-Newtonian behavior whereas for the non-settling slurries, the behavior of the slurry solution depends upon the concentration & physical characteristics of the solid phase within the solution. This naturally affects the flow instability of the settling as well as non-settling slurries.

2. Studies on the Newtonian and non-Newtonian flow transition

Various researchers have considered the following various aspects for the flow transition studies:

(a) Mechanism of the transition related to initiation and propagation of the disturbance (Nouar and Frigaard [39], Frigaard and Nouar, [25], Priymak and Miyazaki [45], Rudman and. Blackburn [47], Guzel [43], Esmael et al. [24]); Avila et al. [1], Bahrani and Nouar [2] etc.);
(b) Criteria of the transition (Dodge and Metzner [9], Escudier et al. [23], Peixinho [41,42], Wilson and Thomas [64], Guzel et al. [29,30], Guzel [43], Japper-Jaafar et al. [33], Esmael et al. [24] and Bahrani and Nouar [2] etc.);
(c) Theoretical, experimental and computational methods for the study (Shan et al. [53], Priymak and Miyazaki [45], Reuter and Rempfer [46], Wilson and Thomas [64], Guzel [43], Bahrani and Nouar [2] etc.);
(d) Effect of fluid property especially rheology (Berman [5], Pinho and Whitelaw [44], Draad et al. [19], White & Mungal [63], Guzel [43] and Guzel et al. [29,30] etc.);
(e) Effect of flow geometry (Slatter [56], Slatter and Wasp [55], Wilson and Thomas [64], Dou et al. [15,16] and Guzel [43] etc.);
(f) Effect of the presence of the multiphase present in the fluid.

Various criteria have been very popular in determining and ascertaining the parameters to check the initiation of the transition in the flow of various kinds of the fluids such as Newtonian, non-Newtonian or any class of the multiphase flow. These various criteria are; Reynolds number based criteria, Fanning friction factor, critical ratio of the energy generation with energy dissipation, asymmetry of the velocity profile, relationship between the pressure drop and velocity ratio etc. Through these parameters or criteria one can check the onset of the transition in the flow.

Various theoretical methods which have been carried out by the various researchers include the linear stability methods, nonlinear stability methods, energy methods etc. Hydrodynamic stability theory is concerned with the response of a laminar flow to a disturbance of small or moderate amplitude. If the flow returns to the original laminar state, the flow is considered to be stable whereas if the flow disturbances grows and causes the laminar flow to change into a different state, the flow can be called as unstable flow. Stability theory deals with the mathematical analysis of the evolution of disturbances superimposed on a laminar base flow. Increased instability leads ultimately to the unstable flow.

A harmonic perturbation is introduced into the linearized Navier-Stokes equations leading to an eigenvalue problem posed as a system of partial differential equations with respect to the spatial coordinates. If a non-zero solution perturbation exists, it is referred to an eigenmode. If the non-zero solution increases with respect to time, the basic flow is said to be linearly unstable whereas if it decreases, the basic flow is said to be linearly stable. Only some of the eigen modes depending on frequencies and wavelengths of the perturbation are amplified. Then, the mathematical form of the perturbation can be prescribed with the mean flow velocity acting as a coefficient.

Although a linear equation governing the evolution of disturbances is desirable, as the disturbance velocities grow above a few percent of the base flow, nonlinear effects become important and the linear equations no longer accurately predict the disturbance