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# Vortex behavior of the Oldroyd-B fluid in the 4-1 planar contraction simulated with the streamfunction–log-conformation formulation

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ABSTRACT

In this paper, we present numerical solutions of the Oldroyd-B fluid flowing through a 4:1 planar contraction, for Weissenberg numbers (*Wi*) up to 20. The incompressible viscoelastic flows are simulated with the streamfunction-log-conformation methodology. The log-conformation representation guarantees by construction the positive-definiteness of the conformation tensor, which circumvents the appearance of the high Weissenberg number problem. In addition, the streamfunction flow formulation removes the pressure variable from the governing equations and automatically satisfies the mass conservation. Thus, the streamfunction–log-conformation reformulation is beneficial for the accuracy and stability of the numerical algorithm. The resulting governing equations are solved with a high-resolution finite-volume method.

Our numerical results for the reattachment length and the intensity of the recirculation vortices produced at the contraction plane are in excellent agreement with the benchmark solutions, available in the literature for Weissenberg numbers up to 3. For highly elastic flows, our results agree qualitatively well with the data of Afonso et al. (2011) [53]. Our simulations predict the reduction of the vortex size with increasing *Wi*, up to $Wi \approx 5$. Moreover, we observe a periodic third vortex growth and annihilation regime for $Wi \geq 15$. The periodic vortex growth and annihilation is correlated with the accumulation of elastic strain in the cavity upstream of the contraction. This elastic instability is viewed as a mechanism that releases the elastic energy accumulated in the Oldroyd-B fluid at the fringe of the recirculation vortices. The dimensionless period of the third vortex annihilation appears to be independent on the Weissenberg number.



## 1. Introduction

Viscoelasticity is a common phenomenon in complex fluids such as polymer melts, polymer solutions and colloidal suspensions, which are widely used in polymer processing (e.g. extrusion, blow molding, injection molding), biofluidics (e.g. vascular flows) and microfluidics (e.g. inkjet printing, lab-on-a-chip). The viscoelastic constitutive behaviors affect several aspects of the flows, including the flow stability, hydraulic resistance, transport efficiency, mixing performance, energy dissipation, etc. In addition, viscoelasticity can trigger elastic instabilities [1,2] and elastic turbulence [3,4], possibly leading to chaotic flow regimes at low Reynolds numbers [5]. Thus, the modeling of viscoelastic flows has a scientific interest and technical perspectives. However, it is considered as a notoriously difficult problem, and to date, numerical simulations are often limited to applications with relatively low levels of elasticity, i.e. where the flows are dominated by viscous effects.

Viscoelasticity typically arises from inter- and intra-molecular interactions of the polymer chains, and depend on the molecular mass and the polymer architecture (e.g. linear chains, branched, etc.). The stress relaxation is explained on the microscale level by kinetic theories [6–8] for diluted, entangled, and crosslinked polymer chains. On the macroscale level, various constitutive models have been derived from idealization and closure approximations of the kinetics models; see for instance [9–11]. The numerical solution of the viscoelastic constitutive models is a difficult task, due to the numerical instabilities that are prone to occur when elastic effects dominate. This issue is referred to in the literature as the *high Weissenberg number problem* [12,13], since the Weissenberg number (*Wi*) is a non-dimensional number quantifying the effect of elasticity on the flow. At large levels of elasticity, the stress field often contains large stress gradients near walls, and stress singularities at sharp corners and stagnation points [14]. Consequently,

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viscoelastic flow solvers are prone to introduce numerical errors because of an under-resolution of the stress field. A turning point is reached when the accumulation of numerical errors alters the positive-definiteness of the conformation tensor (a tensorial variable representing the internal elastic strain [15]), leading to a non-physical state which produces the loss of evolution of the constitutive equation [16] and a numerical divergence [17–20]. Hulsen et al. [21] explained the numerical divergence by the under-estimation of the stress gradients and hence the convective stress fluxes, which are numerically compensated by spurious multiplication of the stress growth rate.

A breakthrough in the simulation of viscoelastic flows was introduced by Fattal and Kupferman [22,23] who reformulated the differential constitutive models in terms of the matrix-logarithm of the conformation tensor—also referred to as the log-*conformation representation*—in order to enforce by construction the positive-definiteness of the conformation tensor. The log-conformation representation also linearizes the exponential stress profiles near the stress singularity, which is beneficial for the accuracy of the numerical scheme. Vaithianathan and Collins [24] introduced two other transformations, based on the eigendecomposition and the Cholesky decomposition of the conformation tensor, which also guarantee its positive-definiteness. Another transformation preserving positive-definiteness, in terms of the square-root of the conformation tensor, was independently introduced by Lozinski and Owens [25] and Balci et al. [26]. Later, Afonso et al. [27,28] generalized these methods under the generic kernel-conformation transformation. Finally, Saramito [29] and Knechtges et al. [30,31] recently derived fully-implicit versions of the log-conformation formulation that do not involve an algebraic decomposition of the velocity gradient tensor, and which can be linearized and solved with the Newton-Raphson method. All these reformulations of the constitutive models significantly improved the robustness of the numerical simulations of viscoelastic flow, and made it possible to simulate viscoelastic flows dominated by elastic effects, at large *Wi*.

Continuity is another constraint that must be satisfied by incompressible fluid flows. Within the classical velocity-pressure flow formulation, the continuity constraint applies to the incompressible flow via the pressure field. In this case, the pressure variable does not hold any thermodynamic information (in contrast to compressible flows), and a pressure (or pressure-correction) equation is derived from the coupling of the conservation of the mass and momentum. Perot [32] has shown that projection (or fractional-step) methods, which decouple the velocity and pressure calculations, introduce decoupling errors that affect the conservation properties of the numerical schemes. The magnitude of the decoupling errors increases with the discrete time-step size and the inverse of the Reynolds number [32]. In addition, Xue et al. [33] tested several semi-implicit velocity-pressure decoupled methods (SIMPLE, SIMPLER, PISO and SIMPLEST), for low-Reynolds number viscoelastic flows, and showed that the overall accuracy and stability of transient calculations highly depend on the chosen decoupling algorithm. Their results highlight that the numerical stability of velocity-pressure decoupling algorithms imposes restrictions on the time-step size.

Comminal et al. [34,35] combined the log-conformation representation with the *streamfunction flow formulation* [36–42], which removes the pressure unknown and automatically fulfills the continuity constraint. The streamfunction formulation was introduced as a change of variable in the vorticity transport equation, in terms of a vector potential of the flow, i.e. the streamfunction. The Kutta-type condition at the interior boundaries of non-simply connected domains is then replaced by constant streamfunction boundary values [43]. This reformulation has also been referred to as an exact projection method by Chang et al. [43], and was extended to three-dimensional parallel calculations in [44]. The streamfunction formulation is formally more accurate than the classical decoupled velocity-pressure methods, since it does not introduce any decoupling error [43]. Moreover, the absence of pressure calculation alleviates the restriction on the time-step size and removes one loop of iterations from the iterative stress-flow solver. Comminal et al. [35] achieved stable calculations of viscoelastic flows in the lid-driven cavity, for Courant numbers as high as 64. Hence, the streamfunction–log-conformation formulation improved the robustness of the numerical simulations, as it enhances at the same time accuracy, stability and iterative convergence.

The 4:1 planar contraction is a geometry that has been used to benchmark the performance of various two-dimensional finite-volume [45–53], finite-element [54–67], spectral-element [68] and hybrid [69–72] viscoelastic flow solvers. In the context of the finite-volume method, Alves et al. [50] have demonstrated the importance of using high-resolution schemes, and proposed the CUBISTA interpolation scheme [73] to enhance numerical accuracy and iterative convergence of the numerical solutions. In spite of the simple geometry, the abrupt planar contraction presents a stress singularity at the reentrant (salient) corner, which makes calculations at high Weissenberg numbers challenging. The contraction flows of viscoelastic liquids have also been the subject of many experimental investigations in planar, axisymmetric and three-dimensional square-square geometries; see for instance [74–80]. The entry flows of Boger fluids generally exhibit a vortex enhancement behavior with increasing Deborah numbers (which measure the ratio between the fluid relaxation and the flow time scales), and in some cases, the formation of an elasticity-driven vortex at the lip entrance [74,78]. Experiments also show that, above a critical Deborah number, the viscoelastic entry flows become time-dependent, with periodic and aperiodic fluctuations [75,76], and a break in the symmetry of the flow [77].

This paper presents the numerical solutions of the Oldroyd-B fluid flowing inside the 4:1 planar contraction, simulated with the finite-volume method and the streamfunction–log-conformation formulation. We report data of the reattachment length and the vortex intensity of the vortices generated on the contraction plane, which we also compare with other results available in the literature. The remainder of the paper is organized as follows: Section 2 presents the governing equations of the viscoelastic flow, as well as the log-conformation representation and the streamfunction reformulation. Details of the numerical method that is used to discretize and solve the governing equations are provided in Section 3. The problem specifications of our simulations of the planar contraction flow are given in Section 4. Numerical results are presented and discussed in Section 5. Finally, the conclusions are drawn in the last section of the paper.

## 2. Governing equations

### 2.1. Standard formulation

The incompressible viscoelastic flow is governed by the conservation of mass (continuity equation) and momentum:

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = -\nabla p + \nabla \cdot \boldsymbol{\tau}, \tag{2}$$

where $\mathbf{u}$ is the velocity vector, $\rho$ is the density of the fluid, $p$ is the isostatic pressure and $\boldsymbol{\tau}$ is the constitutive deviatoric stress tensor.

For the sake of benchmarking the streamfunction–log-conformation formulation, we consider the case of a single-mode quasi-linear viscoelastic liquid. We employ the Oldroyd-B constitutive model [81], which is a continuum model derived from a