Particuology xxx (2016) xxx–xxx

Contents lists available at ScienceDirect

Particuology

journal homepage: www.elsevier.com/locate/partic

Effect of aspect ratio on triaxial compression of multi-sphere ellipsoid assemblies simulated using a discrete element method

Jian Gong, Jun Liu*

State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, Dalian 116024, China

ARTICLE INFO

Article history:
Received 26 February 2016
Received in revised form 24 June 2016
Accepted 12 July 2016
Available online xxx

Keywords:
Ellipsoids
Aspect ratio
DEM
Triaxial compression
Particle shape
Macro-micro response

ABSTRACT

Here, we present a numerical investigation of the mechanical behavior of ellipsoids under triaxial compression for a range of aspect ratios. Our simulations use a multi-sphere approach in a three-dimensional discrete element method. All assemblies were prepared at their densest condition, and triaxial compression tests were performed up to extremely large strains, until a critical state was reached. The stress–strain relationship and the void ratio–strain behavior were evaluated. We found that the stress–dilatancy relationship of ellipsoids with different aspect ratios could be expressed as a linear equation. In particular, the aspect ratio influenced the position of the critical state lines for these assemblies. Particle-scale characteristics at the critical state indicate that particles tend to be flat lying, and the obstruction of particle rotation that occurs with longer particles affects their contact mechanics. Lastly, anisotropic coefficients related to aspect ratio were investigated to probe the microscopic origins of the macroscopic behavior. A detailed analysis of geometrical and mechanical anisotropies revealed the microscopic mechanisms underlying the dependency of peak and residual strengths on aspect ratio.

Introduction

Ellipsoids provide a good model to investigate the effects of particle shape on the mechanical behavior of granular materials (Lin & Ng, 1997; Ng, 2004, 2009). Previous studies (Bagherzadeh-Khalkhali & Mirghasemi, 2009; Zhou, Ma, Chang, & Zhou, 2013) confirmed that ellipsoid-shaped particles offer a realistic representation of soil behaviors. Ellipsoid-shaped particles can be characterized in terms of their aspect ratio, which is defined as the ratio of their major axis to their minor axis. Furthermore, aspect ratio is an important shape factor used to quantitatively describe the particle shape of actual granular materials (Altuhafi, O'Sullivan, & Cavarretta, 2013; Stahl & Konietzky, 2011). Using discrete element methods (DEMs; Cundall & Strack, 1979), two different initial conditions have been used for investigating the shape effects of ellipsoid-shaped particles in previous studies. These studies used either the same void ratio (e) (Ng, 2004; Ting, Meachum, & Rowell, 1995; Yan, 2009; Yang, Wang, & Cheng, 2016), or the same relative density (Azema & Radjai, 2010, 2012; Lin & Ng, 1997; Rothenburg & Bathurst, 1992). Assemblies of ellipsoid-shaped particles with different aspect ratios can have the same e; however, their relative densities will be different because of differences in their limiting densities. Thus, differences in these two initial conditions can lead to conflicting observations. For example, Ng (2009) simulated mono-disperse ellipsoids with different aspect ratios at the same e; he reported that the peak friction angles of the ellipsoids decreased slightly with increasing aspect ratio. However, when all of the assemblies were prepared at their densest condition, Rothenburg and Bathurst (1992) determined that the peak friction angle exhibited a unimodal feature, with varying aspect ratio. Most of the previous DEM simulations investigated the effects of aspect ratio on the mechanical behavior of ellipsoid-shaped particles at the same e, even though a relative density condition is more relevant to laboratory experiments. Alternatively, previous simulations prepared at the same relative density, were conducted in a 2D particle system. In 2D systems, the magnitudes of shear strength and deformation do not agree with those of actual granular materials because of their planar nature. Moreover, most of the earlier studies focused on the effects of aspect ratio on critical state behavior (e.g., Azema & Radjai, 2010, 2012). However, large strain regimes are less relevant to geotechnical applications because of the magnitude of the deformation. In contrast, a relatively small shear strain regime is of great importance because dilatancy and peak shear strength

---

* Corresponding author.
E-mail addresses: gjdlut@mail.dlut.edu.cn (J. Gong), junliu@dlut.edu.cn (J. Liu).

http://dx.doi.org/10.1016/j.partic.2016.07.007

Please cite this article in press as: Gong, J., & Liu, J. Effect of aspect ratio on triaxial compression of multi-sphere ellipsoid assemblies simulated using a discrete element method. *Particuology* (2016), http://dx.doi.org/10.1016/j.partic.2016.07.007

Nomenclature

Symbols

a_c	Anisotropic coefficient of the contact normal
a_{ij}^{c}	Second-order fabric anisotropy tensor
a_{dn}	Anisotropic coefficient of the branch vector
a_{ij}^{dn}	Second-order anisotropy tensor of the branch vector
a_n	Anisotropic coefficient of the normal contact force
a_{ij}^{n}	Second-order anisotropy tensor of the normal contact force
a_t	Anisotropic coefficient of the tangential contact force
a_{ij}^{t}	Second-order anisotropy tensor of the tangential contact force
a_*	Anisotropic coefficients a_c, a_n, a_t, or a_{dn}
a_{ij}^{*}	Anisotropy tensor a_{ij}^{c}, a_{ij}^{n}, a_{ij}^{t}, or a_{ij}^{dn}
D	Diameter of a reference spherical particle having the same volume with ellipsoids
$\bar{d}_0$	Average branch vector calculated for the entire range of Ω
d^c	Branch vector joining the centers of the two particles in contact
$\bar{d}_{ij}'$	Deviatoric part of average branch vector tensors
e	Void ratio
e_c	Critical void ratio
E_c	The contact effective modulus
$\mathrm{E}(\Omega)$	A probability distribution function of Ω
$\bar{f}_0$	Average normal contact force calculated for the entire range of Ω
f^c	Contact force at a given contact
f_n	Normal contact force at a specific contact
f_t	Tangential contact force at a specific contact
F_{ij}^{n}, $F_{ij}^{n'}$	Average normal contact force tensors and its deviatoric part
F_{ij}^{t}, $F_{ij}^{t'}$	Average tangential contact force tensors and its deviatoric part
h	Height of the assembly at a given deformation state
h_0	Initial height of the assembly after isotropic compression
k	Line-fitting parameter
k_n	Normal contact stiffness of the particles
k_s	Shear contact stiffness of the particles
l_0	Initial length of the assembly after isotropic compression
n, n_i, n_j	Unit contact normal vector and its component in the i, j direction
N_c	Number of contacts in particle system
N_p	Number of ellipsoids in particle system
p'	Effective mean stress
q	Deviatoric stress
r	The smaller value of r_a and r_b
r_a, r_b	The radii of particles in contact
S	Scalar parameter
S_c	The sliding ratio
S_r	Normalized quantity of the double contraction of a_{ij}^{*} and σ_{ij}'
t_i, t_j	Component in the i, j direction for unit contact tangential vector
v	Volume of the assembly at a given deformation state
v_0	Initial volume of the assembly after isotropic compression
w	Average rotation of particles about their minor axis
w_0	Initial width of the assembly after isotropic compression
w_i	Rotation of the major axis of the ith particle from the initial state to steady state with $\varepsilon_1 = 65\%$

Greek symbols

ε_1	Axial strain
ε_v	Volumetric strain
μ_b	Inter-particle friction
ϕ	Internal angle of friction
ϕ_{ij}, ϕ_{ij}'	Fabric tensor and its deviatoric part
ϕ_p	Peak internal friction angle
ϕ_c	Residual friction angle
Ω	The orientation of the branch-vector in spherical coordinates
θ_i	The angle between the major axis of the ith particle and the vertical direction
ρ_p	Density of the assembly at peak state
ρ_c	Density of the assembly at critical state
σ_1	Axial stress
σ_2, σ_3	Lateral stresses
σ_c	Confining pressure
σ_{ij}, σ_{ij}'	Stress tensor and its deviatoric part
ψ	Dilatancy angle
ψ_p	Peak dilatancy angle

are relevant to many engineering designs. In this context, we use a 3D particle system, which provides more accurate and reliable results than a 2D particle system, to investigate the effects of aspect ratio on the mechanical behavior of ellipsoids.

Eight types of ellipsoid-shaped particles, generated using a multi-sphere approach, were evaluated in this study. All of the assemblies were prepared at their densest condition for a given generation procedure. Quantitative relationships between particle shape and strength indices (i.e., friction angle and dilation angle), as well as stress–dilatancy relationships were investigated. In addition, the relationships between aspect ratio and particle-scale characteristics, such as coordination number (*CN*), particle rotation, orientational order, percentage of sliding contacts, and contact forces, were studied. Moreover, the accuracy of the stress–force–fabric based on the branch vector frame was analyzed, and any fabric anisotropies affected by aspect ratio were evaluated at peak and residual states. This detailed analysis of fabric anisotropies allowed us to determine the microscopic mechanisms underlying the dependence of shear strength on aspect ratio.

Modeling using the discrete element method

Simulation parameters and assembly generation

The popular DEM program PFC^{3D} (Itasca, 2014) was used for numerical simulations in this study. The ellipsoid-shaped particles were modeled using a multi-sphere (MS) approach. The particle-forming method described by Taghavi (2011) is a built-in function of PFC^{3D} and used to determine the positions and radii of the constitutive spheres, forming each particle. Eight types of particles with different aspect ratios were considered in this study, having aspect ratios of 1.00, 1.05, 1.10, 1.25, 1.50, 1.75, 2.00, and 2.50 (Fig. 1). All ellipsoids had the same volume, equal to the volume of a sphere with a diameter $D = 1.86$ mm (Parafiniuk, Molenda, & Horabik, 2014). Taghavi (2011) defined a distance that corre-

Please cite this article in press as: Gong, J., & Liu, J. Effect of aspect ratio on triaxial compression of multi-sphere ellipsoid assemblies simulated using a discrete element method. *Particuology* (2016), http://dx.doi.org/10.1016/j.partic.2016.07.007