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Comput. Methods Appl. Mech. Engrg. 197 (2008) 309–318

Computer methods in applied mechanics and engineering

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X-FEM a good candidate for energy conservation in simulation of brittle dynamic crack propagation

A. Combescure [a,*], A. Gravouil [a], D. Grégoire [a], J. Réthoré [b]

[a] LaMCoS, INSA-Lyon, CNRS UMR5259, F69621, France
[b] Aerospace Engineering Department, Tu-Delft, Netherlands

Received 9 November 2006; received in revised form 27 March 2007; accepted 4 April 2007
Available online 23 May 2007

Abstract

This paper is devoted to the simulation of dynamic brittle crack propagation in an isotropic medium. It focuses on cases where the crack deviates from a straight-line trajectory and goes through stop-and-restart stages. Our argument is that standard methods such as element deletion or remeshing, although easy to use and implement, are not robust tools for this type of simulation essentially because they do not enable one to assess local energy conservation. Standard cohesive zone models behave much better when the crack's path is known in advance, but are difficult to use when the crack's path is unknown. The simplest method which consists in placing the cohesive segments along the sides of the finite elements leads to crack trajectories which are mesh-sensitive. The adaptive cohesive element formulation, which adds new cohesive elements when the crack propagates, is shown to have the proper energy conservation properties during remeshing. We show that the X-FEM is a good candidate for the simulation of complex dynamic crack propagation. A two-dimensional version of the proposed X-FEM approach is validated against dynamic experiments on a brittle isotropic plate.

Keywords: Dynamic crack propagation; X-FEM; Cohesive zone; Remeshing; Simulation; Crack arrest

1. Introduction

The calculation of dynamic crack propagation remains a difficult challenge. Many contributions have been made on this topic. For the mechanical part, one may mention the works of Freund [14], Bui [19] and Lemaitre [15]. For the computational aspects, many authors have also addressed the problem using different methods, such as local smeared cracking, which relies on material models which include damage [7,6,10], or cohesive zone models, which are clearly related to fracture mechanics concepts and have been proven effective for localized fracture Falk [30–32]. Cohesive zone models have been used extensively, especially in cases where the crack's trajectory is known in advance, and more recently have also been extended to adaptive calculations in which cohesive elements are inserted into the mesh progressively as the crack travels or branches [33–35,28,29]. X-FEM simulation of dynamic crack propagation was first presented by Krysl and Belytschko [25]. The present paper focuses on the comparison of standard FEM dynamic crack propagation simulation with X-FEM simulation. Experimental results are used to assess the validity of the calculations. First, the paper presents the computational models commonly used for crack propagation. We introduce the global theory of dynamic rupture, based on the evaluation of stress intensity factors, followed by the local approach to rupture. Next, we present three usual calculation strategies for the simulation of dynamic crack propagation: element deletion, remeshing, and the use of cohesive zone elements. Then, we briefly present the X-FEM formulation and compare it to the other methods using the same DCB example. We explain the good quality of the dynamic crack propagations obtained using the X-FEM by applying the conservation of energy principle and proving mathematically that the X-FEM method

* Corresponding author. Tel.: +33 4 72 43 64 26.
E-mail address: alain.combescure@insa-lyon.fr (A. Combescure).

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doi:10.1016/j.cma.2007.04.011

guarantees exact energy conservation when the crack propagates. This proof is also valid for the adaptive cohesive zone formulation of dynamic fracture problems using constant strain finite elements. Finally, we apply the X-FEM to the prediction of a crack's propagation in a simple experiment involving a complex crack path with kinks and a stop-and-restart history.

2. Mechanical modeling of dynamic crack propagation

2.1. Global and local approach of rupture

Even though the simulation of dynamic brittle crack propagation remains a difficult challenge, the underlying physical fracture mechanics model is relatively simple and based on three key concepts [19]:

(1) an equation which gives the crack propagation direction;
(2) a criterion for the initiation of crack propagation;
(3) an equation which gives the crack's velocity.

Usually, two approaches are in competition for this type of prediction: a global energy approach, which is often preferred for brittle rupture, and an approach based on local stresses, the latter more effective for ductile fracture. We will limit ourselves to crack propagation driven by the maximum hoop tensile stress alone. Let us recall the main features of the two approaches.

2.1.1. The global energy approach to rupture

Brittle crack propagation is assumed to be governed by the maximum value of the hoop stress $\sigma_{\theta\theta}$ near the crack's tip, which is evaluated using the stress intensity factor $k_{\theta\theta}$:

$$k_{\theta\theta} = \lim_{r\to 0}\sqrt{2\pi r}\,\sigma_{\theta\theta}, \tag{1}$$

where (r,θ) are the local polar coordinates of the crack's tip.

The maximum hoop stress intensity factor and the corresponding local polar angle are denoted K^* and θ^* respectively:

$$K^* = \max_{\theta\in]-\pi,\pi[} k_{\theta\theta} = k_{\theta^*\theta^*} \tag{2}$$

The propagation begins when the maximum hoop stress intensity factor is greater than a critical value called the dynamic crack initiation toughness. The direction of propagation is that of the maximum hoop stress [27]. This criterion can be written as follows:

$$\begin{aligned} &K^* < K_{1d} \quad \text{(no initiation)},\\ &K^* = K_{1d}, \quad \theta^* = \theta_c \quad \text{(initiation)}. \end{aligned} \tag{3}$$

The dynamic crack initiation toughness is a material property which is obtained from experiments.

During the dynamic growth of the crack, the velocity of the crack's tip $\dot{a}$ adjusts itself so that the current maximum hoop stress intensity factor K^* remains equal to the dynamic crack growth toughness:

$$K^*(t,\dot{a}) \geqslant K_{1d} \Rightarrow K^*(t,\dot{a}) = K_{1D}(\dot{a}) \quad \text{(propagation)}. \tag{4}$$

The evaluation of $K_{1D}(\dot{a})$ was given by Kanninen [26], who replaced the quasi-static toughness by the dynamic crack initiation toughness. Then, the dynamic crack growth toughness is assumed to be

$$K_{1D}(\dot{a}) = \frac{K_{1d}}{1-\left(\frac{\dot{a}}{c_R}\right)}. \tag{5}$$

In Eq. (5), c_{R} is the velocity of the Rayleigh waves. Bui [19] calculated the propagation direction θ^* analytically through the following equation:

$$\theta^* = 2\arctan\left[\frac{1}{4}\left(\frac{K_1^{\mathrm{dyn}}}{K_2^{\mathrm{dyn}}} - \mathrm{sign}\left(K_2^{\mathrm{dyn}}\right)\sqrt{8+\left(\frac{K_1^{\mathrm{dyn}}}{K_2^{\mathrm{dyn}}}\right)^2}\right)\right]. \tag{6}$$

The corresponding K^* value is

$$K^* = \cos^3\left(\frac{\theta^*}{2}\right)K_1^{\mathrm{dyn}} - \frac{3}{2}\cos\left(\frac{\theta^*}{2}\right)\sin(\theta^*)K_2^{\mathrm{dyn}}. \tag{7}$$

These physical crack propagation laws provide the key to the prediction of the change in the crack's length at each time step of a transient analysis as proposed by Bui, Freund, or Tuler [19,14,21]. These laws can also be used, when the crack is meshed explicitly or using cohesive zone models, to detect whether it should be remeshed. The calculation of the dynamic stress intensity factors K_1^{dyn} and K_2^{dyn} is necessary for this approach.

2.1.2. Approach based on the local stress and damage

In local fracture models, one dismisses the previous concepts and uses an equivalent method based on the stress and damage fields. The stress and damage states at the crack's tip [15] define how the crack progresses:

(1) the crack propagates if the maximum hoop stress at the crack's tip is greater than a critical value or if damage reaches the critical value;
(2) the crack propagates in the direction of the maximum hoop stress;
(3) in general, the crack propagation velocity is not controlled.

However, this common method based on a local vision of fracture is mesh-dependent: the finer the mesh, the faster the crack's propagation. In order to model brittle failure, one introduces a simple elastic softening failure law.

Then, one calculates the principal stresses at all the integration points of all the elements. If one of these principal tensile stresses exceeds the failure stress σ_c, damage starts to grow. A typical stress–strain curve is shown in Fig. 1, where one can also observe the effect of damage on the