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X-FEM explicit dynamics for constant strain elements to alleviate mesh constraints on internal or external boundaries

P. Rozycki [a,*], N. Moes [a], E. Bechet [c], C. Dubois [a,b]

[a] *Institut de Recherche en Génie Civil et Mécanique, UMR CNRS 6183, Ecole Centrale de Nantes, 1 rue de la Noë, BP 92101, F-44321 Nantes Cedex 3, France*
[b] *PSA – Peugeot Citroën, DRIA/SARA/STEO/MAST, Route de Gisy, Bât. 62 – 1, 78943 Velizy, Villacoublay, France*
[c] *Laboratoire de Physique et Mécanique des Matériaux, UMR CNRS 7554, Université de Metz, Ile du Saulcy, F-57045 Metz Cedex 1, France*

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Abstract

This paper deals with the use of the extended Finite Element Method (X-FEM) for rapid dynamic problems. To solve the equations of motion, a common technique is the explicit direct integration with a Newmark scheme. Since this temporal scheme is only conditionally stable, the critical time step must be determined. It is generally induced by mesh constraints. The idea of the paper is to weaken constraints on mesh generation algorithms so that the critical time step is as large as possible. Using the X-FEM one allows a non-conformity between mesh and discontinuities such as cracks, holes or interfaces. In a first part, we present a summary about direct integration schemes and about the eXtended Finite Element Method. Then, we focus on the theoretical description of a 1D X-FEM finite element and its generalization to 2D and 3D finite elements. Then, dynamic numerical simulations are shown. They concern structures under impact with holes or external boundaries not exactly matched by the mesh. Comparisons are made with numerical results coming from the ABAQUS software. It shows that developments are satisfactory. We conclude with some outlooks concerning this work.

Keywords: X-FEM; Explicit schemes; Dynamics; Holes; Non-conforming meshes

1. Introduction

The complexity of current numerical simulations in rapid dynamics leads to problems with very high CPU time requirements and often cumbersome pre- or post-processing steps. Direct integration methods that are used to solve the equations of motion require relatively small time steps in order to keep the stability of the temporal scheme. Theoretically, the critical time step is based on the highest pulsation of the structure which, in practice, depends on the smallest finite element within the discretized model (and its constitutive law). Often, mesh constraints related to the rigorous respect of complex surface geometries produce "small" finite elements. Even if the mesh can be optimized later on in order to make it more uniform, the procedure can be difficult and may not succeed despite of the progress of meshing tools.

The objective of this paper is therefore to avoid such meshing constraints and to have an easier control on the size of the elements. For this reason, the work is based on developments of the eXtended Finite Element Method (X-FEM), which has been successfully applied to static problems exhibiting discontinuities or heterogeneities such as cracks, holes or material interfaces. The governing idea of this method is to enrich the classical FEM approximation thanks to the Partition of Unity technique with specific functions representing surfaces of discontinuities or heterogeneities. Level sets are used to locate the physical surfaces on the mesh. Their sign indicate the side on which a point is located. Level sets use node-valued functions and are interpolated with the basis functions of the finite element. This description allows to release the underlying mesh from the

* Corresponding author. Tel.: +33 0 2 40 37 68 21; fax: +33 0 2 40 37 25 73.
E-mail address: patrick.rozycki@ec-nantes.fr (P. Rozycki).

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description of surfaces of discontinuity or external boundaries.

As previously described, the X-FEM allows one to treat problems showing discontinuities or heterogeneities. Some works have been done for dynamic problems relative to crack propagations: different techniques of enrichment have been developed to take into account the discontinuity produced by a crack. Some discontinuous enrichments can be used such as in [5] for time-dependent problems. Other authors purpose a generalization of X-FEM by introducing enrichment strategies for time-dependent problems [16]. The discontinuities can also be enriched in time as in [15,7]. In this paper, we restrict our work to the treatment of holes and external boundaries. The X-FEM approach is easier to manage in this case since no specific enrichment functions are necessary. A direct use of enrichment functions is problematic for explicit schemes because a lumping technique is not easily available for such approximation. A work in this direction for cracks may be found in [11].

In the first part of this paper, the explicit direct integration method is first summarized: we recall the definition of the critical time step and the definition of stability and dispersion properties. Then we present the X-FEM for static problems. The second part of the paper introduces a 1D X-FEM finite element: we propose here a theoretical approach to treat the presence of void volume within this 1D finite element as well as a specific technique of mass lumping. Results from this simple 1D model are then generalized for triangular (2D) and tetrahedral (3D) finite elements. In the last part of this work, some numerical simulations in 1D, 2D and 3D cases are carried out. They are related to pierced structures under impact loads and are compared to simulations carried out on conforming meshes with the ABAQUS software. We show several results about convergence and stability of the technique. The conclusions drawn from these results finally guides the outlook concerning the X-FEM for rapid dynamics.

2. Foreword about the explicit Newmark scheme and the X-FEM

2.1. Explicit direct integration method

The following set of relations (1) gives the equations of dynamics, under matrix form, for a solid body at the finite element level

$$M\ddot{U} + KU = F^{\text{ext}}. \tag{1}$$

The most general approach to solve (1) in time consists in using direct integration methods because the size of the system is usually very large. Among many techniques, the Newmark scheme [3,4,13] is commonly used for rapid dynamics: for each time step Δt, the accelerations (2) are computed and then allow to update the displacement and velocity fields

$$\ddot{U}_{n+1} = M^{-1}\left(F^{\text{ext}}_{n+1} - K\left[U_n + \Delta t\dot{U}_n + \frac{\Delta t^2}{2}\ddot{U}_n\right]\right). \tag{2}$$

Eq. (2) shows that the explicit Newmark method possesses the advantage in the fact that during the resolution of equilibrium equations, only the mass matrix requires to be inverted. Furthermore, it is possible to apply mass lumping techniques which render this mass matrix diagonal. This leads to a very efficient numerical scheme.

However the explicit Newmark scheme also possesses the disadvantage of the necessity to impose a critical time step in order to maintain its stability. This constraint is also called CFL (Freidrich–Lévy–Courant) condition [3,4,9,13].

Stability is insured when the norm of the greatest eigenvalue of system is strictly inferior to one. Rather than identifying this previous eigenvalue which is quite time consuming, one can use the highest eigenvalue of each finite element taken separately. It is easier and much faster to find the smallest finite element within the most disadvantageous material to impose the critical time step. It can be shown [9] that this time step (3) constitutes a lower bound of the one issued from the complete structure, and that therefore respects the CFL condition. It is also a good approximation if the mesh is almost uniform

$$\Delta t \leqslant \frac{2}{\omega_{\max}} = \Delta t_{\mathrm{C}}. \tag{3}$$

This reveals a potential problem with the explicit approach: if the discretized structure has just one very small finite element, the latter will constrain the critical time step for the simulation of the whole structure. That is the reason why some freedom is offered to the users of commercial software in choosing an "appropriate" time step (possibly bigger) with stabilization techniques. Nevertheless, the choice of the time step is generally limited by instability as upper bound and by the phenomenon of dispersion as lower bound:

- Instability occurs if the time step is greater than the critical time step. Overestimated forces are transmitted and reflected within some finite elements. The process repeats itself at each time step and it quickly leads to the divergence of the numerical scheme.
- Dispersion occurs if the time step is much lower than the critical time step. Small disturbances are generated and they are transmitted with a wave speed higher than physical wave speed. It implies an attenuation of the real efforts. It should be noticed that even if the quality of the solution is somewhat degraded, it is generally accurate. The smallest dispersion is however obtained for a time step equal to the critical time step.

2.2. The X-FEM

Within the finite element method the presence of discontinuities or heterogeneities such as cracks, inclusions or material interfaces constrains the mesh to be in conformity