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A sequential nonlinear interval number programming method for uncertain structures

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ABSTRACT

A sequential nonlinear interval number programming (SNINP) method is suggested to deal with the *uncertain* optimization problems. A general uncertain optimization model is investigated in which the objective function and constraints are both nonlinear and uncertain. A nonlinear interval number programming (NINP) method is employed to transform the uncertain optimization problem into a deterministic two-objective optimization problem. Then using the linear combination method and the constraint penalty function method, a deterministic single-objective and non-constraint optimization problem is formulated in terms of a penalty function. Combining this NINP method with a modified approximation management framework (AMF), an efficient SNINP method is then constructed. At each iterative step, an approximation optimization problem is created based on the Latin Hypercube Design (LHD) and the quadratic polynomial response surface approximation (RSA), and it can then be solved by the NINP method efficiently. The trust region method is used to manage the sequence of the approximation optimization problems based on a reliability index. An efficient method is suggested to calculate the actual penalty function and whereby the reliability index, and based on it the current design space can be updated. Two numerical examples are presented to demonstrate the effectiveness of the present method.

1. Introduction

In practical structures, many kinds of uncertainties are always encountered. Some of them have a physical origin concerned with loading conditions, material properties, etc., and they cannot be influenced by the designer. The other uncertainties are caused by people, and the typical examples are manufacturing and measurement deviations. To obtain a reliable or robust design for uncertain structures, the effective uncertain optimization methods should be researched and developed.

The probability method is widely and successfully used to deal with the uncertainty, and based on it various kinds of stochastic programming methods are constructed [1–6]. Additionally, the structural mechanics was combined with the stochastic programming, and whereby several uncertain structural optimization methods were developed [7–9]. In the above probability-based optimization methods, random variables are used to model the uncertain parameters, and the uncertain optimization problem is generally transformed into a deterministic optimization problem using the probability statistics theory. Using the probability method, a great amount of experimental data are always required to construct the precise probability distributions of the uncertain parameters. Unfortunately, for practical engineering problems, the experimental data available for an uncertain parameter are often not sufficient, and hence the probability method will encounter difficulty. However, depending on the engineering experience, these data are generally enough to identify the bounds of the uncertain parameter [10]. Thus if an uncertain-but-bounded variable can be used to model the uncertainty of a parameter, the uncertainty analysis can hence be made much more convenient and economical.

In the past two decades, the interval method has been obtaining more and more attentions. *Interval* represents a closed bounded set of real numbers, and in interval mathematics [11] it is regarded as a type of *number*, namely *interval number*. Using interval method, the lower and upper bounds of the uncertain parameters are only required, unnecessarily knowing their precise probability distributions. Based on the interval method, another type of uncertain optimization method, namely *interval number programming*, has come into being. Tanaka et al. [12], Ishibuchi and Tanaka [13], Rommelfanger [14] discussed the linear programming problems with interval coefficients in the objective function. Tong [15] considered the case in which the coefficients of the objective function and constraints are all intervals, and the possible interval of the solution was obtained by taking the maximum value range and minimum value range inequalities as constraint conditions. Liu and Da [16] proposed a fuzzy satisfactory degree of interval number to deal

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with the uncertain constraints. Sengupta et al. [17] studied the linear interval number programming problem on the basis of a comparative study on ordering interval numbers. Zhang et al. [18] assumed the interval numbers as random variables with uniform distributions and proposed a possibility degree to solve a multi-criteria decision problem. These works all focus on the linear interval number programming. However, researches on the nonlinear interval number programming (NINP) which is really useful for most of the practical engineering problems are still scarce. On the best knowledge of the authors, Ref. [19] seems the first attempt to study the NINP problem. This reference opens the door for NINP research, however, the uncertain constraints have not been investigated and furthermore the low optimization efficiency blocks its practical applications. This work was improved by the Ref. [20] in which the uncertain constraints were also considered as well as the uncertain objective function. However, the low efficiency problem also exists. Jiang et al. [21–23] employed the interval analysis method to calculate the bounds of the uncertain objective function and constraints at each optimization step, and whereby developed an efficient NINP method. However, this method is only effective for problems, in which the derivatives of the uncertain objective function or constraints with respect to each uncertain parameter can be calculated precisely. As a result, so far an NINP method with high optimization efficiency and wide applicability has still not been developed. In NINP, we need to transform the uncertain optimization problem into a deterministic one, which is generally a nesting optimization problem. The outer optimization is used to optimize the design vector and the inner optimization is used to compute the intervals of the uncertain objective function and constraints, and this generally leads to an extremely low optimization efficiency for a practical engineering problem. Actually, the low efficiency problem is the biggest difficulty existing in the current NINP research, which has been a huge stone to block the development and practical application of the NINP methods.

Today's engineering problems become more and more complex. Many sophisticated analysis and simulation software solvers are adopted in design study. These analysis and simulation processes are usually computation-intensive which take considerable time and bring high computation cost [24]. A common engineering practice is to construct a simple and explicit approximation model to replace the computation-intensive simulation model, and combine the approximation model with a nonlinear optimization operator to obtain a high computation efficiency. The approximation optimization methods have been widely studied and an amount of literatures have been published e.g. [25–30]. In recent years, there has been a growing interest in the approximation management framework (AMF) which combines the approximation optimization with the trust region method. From the viewpoint of mathematics, the approximation optimization based on trust region can ensure convergence to a Karuch–Kuhn–Tucker solution [31,32]. Through the researches on AMF [31–35], two prominent merits of AMF can be found. One is the high optimization efficiency brought by the approximation models, and the other is the adaptive improvement of the optimization precision brought by the trust region method. Thus, it seems promising and inspiring to introduce the AMF into NINP and hence construct an efficient uncertain optimization method. However, all of the above AMFs were developed for deterministic optimization problems. In NINP problems, influences of the uncertain parameters must be considered, and therefore construction of the approximation models and the approximation model management will be much more complex and difficultly treated than deterministic problems. Therefore, a significant modification for the AMF is absolutely necessary to meet the special requirements of the NINP problems.

Concentrating on the major problem existing in the current NINP methods, this paper aims at developing a new NINP method with high optimization efficiency and wide applicability. The following text consists of four major parts. The first part is the statement of the problem, in which an uncertain optimization model is given and introduced. In the second part, an NINP method is used to transform the uncertain optimization problem into a deterministic optimization problem. This part is based on the authors' previous work [20,23]. In the third part, a modified AMF is combined with the NINP method, and hence an efficient sequential nonlinear interval number programming (SNINP) method is constructed. A sequence of approximation optimization problems are generated under the management of the trust region method, and at each iterate the approximation models are constructed for the uncertain objective function and constraints based on the Latin Hypercube Design (LHD) and the quadratic polynomial response surface approximation (RSA). Each approximation optimization problem is solved through the nesting optimization of an inter-generation projection genetic algorithm (IP–GA) [36,37]. An efficient method is provided to compute the actual penalty function and whereby the reliability index, based on which the current design space can be updated. In the forth part, a benchmark test is analyzed to test the performance of the present method, and then the present method is also applied to a practical engineering problem.

2. Statement of the problem

A general structural optimization problem can be formulated as follows:

$$\min_{\mathbf{X}} f(\mathbf{X})$$

subject to

$$\begin{aligned} & g_i(\mathbf{X}) \leqslant b_i, \quad i = 1, \ldots, l, \\ & \mathbf{X}_l \leqslant \mathbf{X} \leqslant \mathbf{X}_r, \end{aligned} \tag{1}$$

where $\mathbf{X}$ is an n-dimensional design vector, and $\mathbf{X}_l$ and $\mathbf{X}_r$ denote the allowable minimum and maximum vectors of $\mathbf{X}$, respectively. f and g_i are objective function and the ith constraint, respectively, and in practical structural analysis they are generally obtained from the simulation models. l is the number of the constraints. b_i is an allowable maximum value of the ith constraint. Supposing that there exits uncertainty in the structure and the interval method is used to describe the uncertainty, a following uncertain optimization problem can be formulated:

$$\min_{\mathbf{X}} f(\mathbf{X}, \mathbf{a})$$

subject to

$$\begin{aligned} & g_i(\mathbf{X}, \mathbf{a}) \leqslant b_i^I = [b_i^L, b_i^R], \quad i = 1, \ldots, l, \\ & \mathbf{a} \in \mathbf{a}^I = [\mathbf{a}^L, \mathbf{a}^R], \ a_i \in a_i^I = [a_i^L, a_i^R], \quad i = 1, 2, \ldots, q, \\ & \mathbf{X}_l \leqslant \mathbf{X} \leqslant \mathbf{X}_r, \end{aligned} \tag{2}$$

where $\mathbf{a}$ is a q-dimensional uncertain vector, and here f and g are required to be continuous with respect to $\mathbf{a}$. The uncertainty of $\mathbf{a}$ is modeled by an interval vector $\mathbf{a}^I$. The superscripts I, L, and R denote interval, lower and upper bounds of interval, respectively. b_i^I is an allowable interval of the ith uncertain constraint. In this optimization model, the uncertainty level, namely intervals of the uncertain parameters are assumed to be relatively small. As f and g are continuous functions of the uncertain parameters, all of the values of the objective function or each constraint under the possible combinations of $\mathbf{a}$ will form an interval, instead of a real number. Thus, Eq. (2) cannot be solved through traditional optimization methods, in which the objective function and constraints are all deterministic values at a specific design vector.