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Stabilized low-order finite elements for coupled solid-deformation/fluid-diffusion and their application to fault zone transients

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ABSTRACT

Finite element simulations of coupled solid-deformation/fluid-diffusion occurring in earthquake fault zones often require high-fidelity descriptions of the spatial and temporal variations of excess pore water pressure. Large-scale calculation of the coupled fault zone process is often inhibited by the high-order interpolation of the displacement field required to overcome unstable tendencies of the finite elements in the incompressible and nearly incompressible limit. In this work we utilize a stabilized formulation in which the balance of mass is augmented with an additional term representing a stabilization to the incremental change in the pressure field. The stabilized formulation permits equal-order interpolation for the displacement and pore pressure fields and suppresses pore pressure oscillations in the incompressible and nearly incompressible limit. The technique is implemented with a recently developed critical state plasticity model to investigate transient fluid-flow/solid-deformation processes arising from slip weakening of a fault segment. The accompanying transient pore pressure development and dissipation can be used to predict fault rupture and directivity where fluid flow is an important driving force.

1. Introduction

In order to accurately model the behavior of fluid-saturated geomaterials, it is necessary to account for the strong coupling between the solid skeleton and pore fluid. This coupling is of particular interest when studying fault zone processes, and is central to many open questions about fault behavior. The presence of fluids might explain why some faults, such as the San Andreas, are weaker than expected [1,2]. Increases in pore pressure may tend to weaken faults by reducing the effective normal stress, and trigger seismic activity. If the overpressures are too large, however, the fault could experience stable, rather than unstable, sliding [3]. Dilatancy or compaction within the fault zone will also play a crucial role, as well as the degree to which fluid exchange is allowed to occur between the fault and its surroundings.

Finite element simulations provide a natural tool for investigating these processes. To do so, we employ a mixed $\boldsymbol{u}/p$ formulation to solve for the solid displacements and fluid pressures. In comparison to pure-displacement formulations, however, the mixed scheme creates additional challenges for the numerical analyst. In the limit of low permeability or fast loading rates, the pore fluid can impose near or exact incompressibility on the deformation of the solid matrix. In the presence of incompressibility constraints, it is well known that only certain combinations of discrete spaces for the pressure and displacement interpolation exhibit stable behavior. Failure to choose a stable pair can lead to poor results, typically in the form of spurious oscillations in the pressure field and sub-optimal convergence behavior.

The same restrictions are found in other constrained problems in solid and fluid mechanics. Classic examples include mixed formulations for Stokes flow, Darcy flow, and incompressible elasticity. The mathematical theory establishing the solvability and stability characteristics of mixed formulations is well-developed. The key ingredients are the ellipticity requirement and the famous Ladyzhenskaya-Babuška-Brezzi (LBB) condition [4,5]. Unfortunately, many seemingly natural interpolation pairs – including equal-order interpolation for all field variables – do not satisfy the necessary stability requirements. In practice, most analysts rely on "safe" elements such as the Taylor-Hood family, in which the displacement interpolation is one-order higher than the pressure interpolation. A variety of more sophisticated stable elements are also available, for example, [6,7].

From an implementation point of view, it would be appealing to circumvent the stability restrictions and employ a broader class of interpolation pairs. Over the years, many stabilization techniques have been proposed for doing precisely this, most extensively in the fluid dynamics community. The model equations used to study these schemes are typically the Stokes or Darcy equations, which despite their simplicity contain all of the salient features of a constrained problem. We mention the early Brezzi-Pitkäranta scheme [8], the Galerkin Least-Squares (GLS) approach of Hughes et al., [9],

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and the more recent Variational Multiscale Methods [10] – but many others exist [11–13]. In solid mechanics, a variety of schemes have also been developed for incompressible and quasi-incompressible elasticity in order to overcome volumetric locking associated with pure-displacement formulations. For example, Oñate et al. [14] proposed a formulation based on the concept of Finite Calculus. Masud and Xia [15,16] developed a formulation for both linear and nonlinear constitutive models based on a Variational Multiscale approach. Romero and Bischoff have recently proposed an interesting method for linear elasticity which involves enriching the finite element spaces with incompatible bubble functions [17]. Of course, the above schemes are merely a representative sample of an extensive literature that has developed for each class of problems.

While it is difficult to classify all stabilization schemes in a unified framework, most frequently the methods lead to a modified variational formulation in which additional terms are added to the mass balance equation, modifying the incompressibility constraint in such a way that stability of the mixed formulation is increased while maintaining a convergent method. In this way, meaningful results can be obtained when using otherwise unstable elements. The goal of this contribution is to extend the stabilization concept to coupled solid-deformation/fluid-diffusion problems. While stabilized methods are employed frequently in fluid and solid mechanics problems, their use in coupled geomechanical problems is limited. Nevertheless, some good work in this direction has begun. In [18], Wan used the GLS approach to stabilize both a displacement–pressure and a displacement–pressure–velocity formulation. In [19,20], Truty and Zimmerman compared three schemes: one based on the Brezzi-Pitkäranta stabilization and two based on the GLS approach. They also extended their formulation to account for partial saturations. In [21,22], Pastor et al. proposed a stabilization scheme for dynamic problems using a fractional-step algorithm, incorporating the stabilization into the time-stepping scheme. In each case, the authors demonstrated that the stabilizations can successfully suppress instabilities and lead to good-quality solutions. Of course, each scheme has its own shortcomings. For example, the GLS method is based on adding the residual of the strong form of the governing equations. As such, second-order derivatives with respect to the displacements appear, and when using linear interpolation, these terms either vanish or are poorly approximated. A special technique must generally be employed to improve the accuracy of these calculations, introducing additional computational work. See [18], for example, where Wan develops such a stress recovery technique. The GLS formulations also often lead to a non-symmetric modification of the system matrix. While this makes little difference if the original problem is non-symmetric, it would be appealing to preserve any symmetry if it does exist. Indeed, a key advantage of the methods of [15–17] is their symmetry-preserving property, but these schemes have only been employed for incompressible solids and have not been extended to coupled solid/fluid formulations. The Brezzi-Pitkäranta scheme does lead to a symmetric modification and can be cheaply implemented for equal-order linear interpolations. Unfortunately, the formulation cannot be extended to stabilize other unstable pairs such as linear-displacement/constant-pressure elements. The fractional-step method is primarily designed for dynamic problems, and may not be an efficient approach for quasi-static models. It also leads to a conditionally stable time-integration scheme even if the underlying algorithm is implicit, though recent improvements by the authors have significantly improved the stability restriction [23].

In this paper, we introduce a new stabilization scheme for coupled geomechanical problems based on the concept of Polynomial-Pressure-Projections. In this approach, the additional stabilizing terms use element-local projections of the pressure field to counteract the inherent instabilities in the chosen interpolation pair. The technique was recently proposed by Dohrmann, Bochev, and Gunzburger, and has been successfully employed for stabilizing the Stokes problem [13,24] and Darcy problem [25]. An analysis of similar pressure projection methods, and a unifying framework for their analysis, has also been proposed by Burman [26].

In this work we employ pressure projections to address instabilities that arise in the geomechanical problems under consideration. The new stabilization has several appealing features. In particular, the additional stabilizing terms can be assembled locally on each element using standard shape function information, and no specialized subroutines are required. The scheme does not require the calculation of higher-order derivatives or special stress-recovery techniques. The method introduces minimal additional computational work, and can be readily implemented in a standard finite element code. The scheme also leads to a symmetric modification of the system matrix, preserving any symmetry that was inherent in the original variational formulation. The resulting method thus shares many of the positive features of the Brezzi-Pitkäranta stabilization, but can be used to stabilized a broader class of unstable pairs.

The primary motivation for using stabilization is computational efficiency. As an example, consider two meshes composed of an equal number of elements. The first mesh employs continuous biquadratic-displacement/bilinear-pressure quads (Q9P4), while the second uses bilinear-displacement/bilinear-pressure quads (Q4P4). Both elements are illustrated in Fig. 1. The first element possesses 22 degrees of freedom and is known to be stable, while the second element has 12 degrees of freedom and is known to be unstable – unless a stabilized formulation is employed. The two elements are comparable in the sense that they produce the same order of pressure interpolation. The Q9P4, however, leads to algebraic problems with many more degrees of freedom. As the number of elements in each mesh grows, a simple argument shows that the total number of unknowns in the two meshes quickly approaches a ratio of 3:1. If we consider the equivalent three-dimensional situation, this ratio approaches $6\frac{1}{4}:1$. The bandwidth of the sparsity patterns will grow similarly.

Further computational savings can also be associated with the quadrature rule employed. The Q9P4 element typically requires 3×3 Gauss-quadrature in order to accurately integrate the quadratic displacement field. In the Q4P4 mesh, we only need 2×2 quadrature. If we consider an elastoplastic material in which a significant level of computation must be performed in the material subroutine at each Gauss point, the lower-order quadrature rule will lead to additional efficiency. The equal-order element can also somewhat simplify the code implementation, particularly when employing adaptive mesh refinement or a parallel decomposition of the domain. Finally we note that the introduction of stabilization terms can often improve the convergence behavior of iterative solvers. For extremely large problems, the memory-efficiency of iterative solvers makes them a more attractive choice than sparse direct solvers. For an extensive discussion of the numerical solution of algebraic systems of the type considered here, see [27].

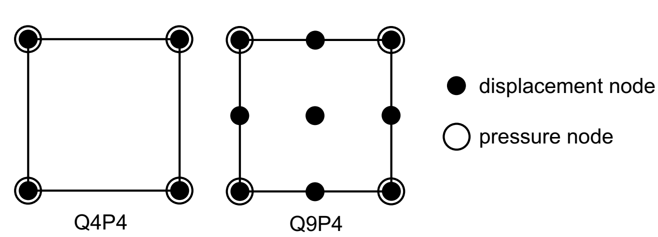

Fig. 1. Example mixed elements, showing the unstable Q4P4 and stable Q9P4.