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Immersed finite element method for rigid body motions in the incompressible Navier–Stokes flow

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Abstract

The immersed finite element method (IFEM) takes places as a method developed for the purpose of solving effectively fluid–structure interaction problems including multi-physics ones. In the original IFEM, the reproducing kernel particle method (RKPM) playing a role of discrete Dirac delta function is employed to distribute the interacting force on the structure to the surrounding fluid and calculate the velocity on the structure induced from the background fluid. In this paper, fluid–structure interaction (FSI) problems in 2D between the incompressible Navier–Stokes flow and rigid structure are considered and we make use of the transformed finite element basis functions to replace discrete Dirac delta functions such as RKPM. This replacement makes the numerical support of FSI force distributed to the fluid smaller than that in case where the discrete Dirac delta function is used. In the finite element formulation, reducing the support size of distributed FSI force affects the accuracy of numerical solutions near the structure. The comparison of our numerical solution for particulate flows shows this fact well. We calculate particulate flows of rigid circular and rod type disks and compare them with the preceding results of reliability to show a good agreement to them. Moreover, the interaction motion for rod type of rigid structure in the fluid is traced.

Keywords: Fluid–structure interaction; Immersed finite element method; Particulate flow; 2D incompressible Navier–Stokes flow

1. Introduction

For a couple of recent decades, although computational power has been drastically increased and efficient numerical methods have developed to solve the fluid–structure interaction (FSI) problems, the computing time, and accuracy still have been issues in this field. FSI becomes one of essential parts in multi-physics. However, because of its complexity, the accurate computation with less time is far off as usual. For instance, conventional methods solve the FSI problem through coupling and merging between fluid and solid at each time step. When the solid moves in fluid, re-meshing parts should be inevitably involved at each time step and it will be mostly a time consuming work in computation.

To overcome this difficulty, immersed type of methods appear, for example, the immersed boundary (IB) method [1], the extended finite element method [2], and the fictitious domain method [3–6]. Especially, IB method was developed at first for simulating the heart. After that, it has been applied to the FSI problems [7–11] and even to complex flows with fixed rigid boundary [12]. More recently, inspired by IB method, the immersed finite element method (IFEM) [13,14], the immersed continuum method (ICM) [15], and the extended immersed boundary method (EIBM) [16] were developed to solve the motion and large deformation of incompressible hyper-elastic material within an

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incompressible (or slightly compressible) fluid. Using the IFEM, problems in nano mechanics and biology are successfully solved: nanowire alignment [17,18], rheology of red blood cell [19], and the other applications [20,21] in biology.

The main advantage of IFEM is to be able to exclude the re-meshing during the simulation of moving structures in fluid, which is a common advantage with IB method. IB methods use the finite difference scheme, while the IFEM employs two kinds of finite element meshes: (1) structure mesh at the initial time and (2) fluid mesh on the entire domain. Therefore, the fluid mesh is of Eulerian type while the structure mesh can be viewed as Lagrangian description in conjunction with the so-called flow map. On the entire domain containing both structure and fluid, the flow and moving structure can be solved by IFEM counting as one continuum which is governed by Navier–Stokes equations with additional momentum force (FSI force). Salient feature only in IFEM is that the reproducing kernel particle method (RKPM) is employed instead of the discrete Dirac function in IB method in order to distribute the FSI force onto the fluid [22].

In this paper, an improved IFEM will be developed based on the directly coupled FSI force treatment, which is associated with transformed finite element basis functions between fluid and structure domains in exchange for discrete Dirac delta functions. This change can be expected to remove the inaccuracy due to the FSI force distribution to the fluid using the discrete Dirac delta function. In theoretical point of view, the support of FSI force is concentrated exactly on the structure domain itself [14]. However, the numerical support of FSI momentum force in IFEM equipped with RKPM as a discrete Dirac delta approximation is larger than that in theory. The transformed finite element basis functions are able to optimally reduce the support size of distributed FSI force in the viewpoint that the support is composed of union of elements.

Therefore, the emphasis in this paper is laid on achieving the optimal support in IFEM by replacing the discrete Dirac delta approximation with the transformed finite element basis functions. The immediate effect of the optimal support is expected to appear in accuracy improvement. To show the accuracy of the proposed method, particulate flows of single disk and two disks are simulated and then we compare the results to the other [6]. In case of two disks, the simple collision model as in [5] is adopted. As an example for asymmetric rigid structure different from disk, a motion of rod shaped rigid body is calculated by our method and the curved trajectory of this body is obtained as we anticipated from the asymmetry.

2. Fluid–structure interaction formulation based on IFEM

IFEM basically stems from IB method. Both IB and IFEM regard the structure as the momentum perturbation in fluid. Throughout the paper, by the entire domain, we mean the domain consisting of the real fluid and structure regions, i.e., the union of both regions. After replacing the structure with equivalent momentum force (FSI force) and taking the corresponding Lagrangian equations for the dynamics of structures, the flow on the entire domain will be iteratively solved for the nonlinearity of FSI problem and then the resultant velocity of the flow on the entire domain makes the structure move. The body force and additional FSI force are treated in an explicit manner.

In this paper, the fluid is assumed to be the incompressible viscous flow and the structure to be rigid body.

2.1. FSI interaction force in IFEM formulation

Let Ω be the entire domain of consideration and Ω_t^{S} the structure domain occupied by the rigid body immersed in the fluid at time t. Then, Ω_0^{S} represents the initial domain for the structure and the pure fluid domain will be $\Omega \setminus \Omega_t^{\mathrm{S}}$ at time t. If a distribution of velocity $\mathbf{u}(\mathbf{x}, t)$ on the entire domain Ω is given, we can define the flow map $\mathbf{x}(\mathbf{X}, t) : \Omega_0^{\mathrm{S}} \to \Omega_t^{\mathrm{S}}$ at time t as trajectories of points starting from Ω_0^{S}:

$$\frac{\partial \mathbf{x}(\mathbf{X}, t)}{\partial t} = \mathbf{u}(\mathbf{x}(\mathbf{X}, t), t), \quad \mathbf{X} \in \Omega_0^{\mathrm{S}}. \tag{1}$$

The flow map above enables us to define the structure velocity $\mathbf{u}^{\mathrm{S}}$ at time t

$$\mathbf{u}^{\mathrm{S}}(\mathbf{X}, t) \equiv \mathbf{u}(\mathbf{x}(\mathbf{X}, t), t), \quad \mathbf{X} \in \Omega_0^{\mathrm{S}}. \tag{2}$$

Then, the system of fluid–structure interaction problem is described as follows:

$$\rho_{\mathrm{f}}\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u}\right) - \mu\Delta\mathbf{u} + \nabla p = \rho_{\mathrm{f}}\mathbf{g}, \quad \mathbf{x} \in \Omega \setminus \Omega_t^{\mathrm{S}}, \tag{3}$$

$$\nabla \cdot \mathbf{u} = 0, \quad \mathbf{x} \in \Omega \setminus \Omega_t^{\mathrm{S}}, \tag{4}$$

$$\rho_{\mathrm{s}}\frac{\partial \mathbf{u}^{\mathrm{S}}(\mathbf{X}, t)}{\partial t} = \rho_{\mathrm{s}}\mathbf{g} + \nabla_{\mathbf{x}} \cdot \boldsymbol{\sigma}^{\mathrm{S}}, \quad \mathbf{X} \in \Omega_0^{\mathrm{S}}, \tag{5}$$

where the structure stress $\boldsymbol{\sigma}^{\mathrm{S}}$ is defined as

$$\boldsymbol{\sigma}^{\mathrm{S}}(\mathbf{X}, t) = -p(\mathbf{x}(\mathbf{X}, t), t)I + \mu[\nabla_{\mathbf{x}}\mathbf{u}(\mathbf{x}(\mathbf{X}, t), t) + (\nabla_{\mathbf{x}}\mathbf{u}(\mathbf{x}(\mathbf{X}, t), t))^{\mathrm{T}}]. \tag{6}$$

From the definition of structure velocity in (2), the no-slip condition between the rigid body and surrounding pure fluid is satisfied automatically. Using the flow map in (1), the Lagrangian equation (5) can be transformed into the Eulerian equation:

$$\rho_{\mathrm{s}}\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u}\right) = \rho_{\mathrm{s}}\mathbf{g} + \nabla_{\mathbf{x}} \cdot \boldsymbol{\sigma}^{\mathrm{S}}, \quad \mathbf{x} \in \Omega_t^{\mathrm{S}}. \tag{7}$$

The momentum force of the incompressible Navier–Stokes equations is modified in IFEM in terms of additional force considering the effect of the moving immersed rigid body as follows: