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Treatment of acoustic fluid–structure interaction by localized Lagrange multipliers: Formulation

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Abstract

A new concept is presented for modeling the dynamic interaction between an acoustic fluid and an elastic structure. The coupling of this multiphysics system is done by inserting a kinematic interface frame between the fluid and the structure, and using node-collocated Lagrange multipliers to connect the frame to each subsystem. The time-domain response analysis is performed by a partitioned analysis procedure. The main advantages of this localized Lagrange multiplier (LLM) primal-dual coupling method are: complete localization of the structure and fluid subsystems, elimination of the conventional predictor in the partitioned time integration method, and the ability to accommodate non-matching meshes. The standard Newmark time integrator is used on both the fluid and structure models. It is shown that if the integrator is A-stable and second-order accurate for a monolithic treatment, it retains those properties for both Mortar and LLM partitioned solution procedures. Infinite and finite piston problems are used to explain and verify the methodology. A sequel paper under preparation presents and discusses a set of benchmark and application examples that involve the response of existing dams to seismic excitation.

Keywords: Coupled systems; Fluid–structure interaction; Earthquake dynamics; Partitioned analysis; Non-matching meshes; Lagrange multipliers

1. Introduction

The dynamic interaction between a fluid and a structure is a significant concern in many engineering problems. These include systems as diverse as aircraft, rockets, turbines, marine structures (fixed, floating and submerged), airbags, parachutes, storage tanks, dams, biomechanical systems, inkjet printers and suspension bridges. The interaction may change the dynamic characteristics of the structure and consequently its response to transient, periodic and stochastic excitation. The model-based simulation of this class of coupled multiphysics systems presents three technical challenges.

The first is *discretization heterogeneity*. Effective space and time discretization methods for the two interacting components are not necessarily the same. This dilemma is particularly pressing when one would like to use available but separate computer codes for the fluid and the structure treated as individual entities, and use them to solve the coupled problem.

The second is effective treatment of the interaction when the discrete structure and fluid meshes *do not match* over the interface. Non-matching spatial meshes may occur for various reasons: a component may require a finer mesh for accurate results; teams using different programs construct or generate the meshes separately; or the discretization of one or both components is determined *a priori* for other reasons, for example incremental simulation of the structure construction process. If different time-stepping schemes are used (for example, implicit in the structure and explicit in the fluid), solutions may not match in time either.

The third is *forestalling performance degradation* in simulations. Even if the separate discrete models are satisfactory

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as regards to stability and accuracy, the introduction of interaction may have damaging effects on the coupled response. Furthermore, if the coupled components have widely different physical characteristics (stiffness, mass density, etc.), the coupled system may be scale-mismatched by orders of magnitude. A poorly scaled discrete model may produce unacceptable errors, particularly under long-term periodic or cyclic loading.

This paper presents the development of a primal-dual coupling method for treating the interaction of an acoustic fluid with a flexible structure, with emphasis on handling spatially *non-matching* meshes. This is called the localized Lagrange multiplier (LLM) method. A *frame* is introduced as "mediator" device between the fluid and the structure over the interaction surface. The frame is discretized in terms of kinematic (primal) variables. A Lagrange multiplier field is introduced between the frame and the structure, and another one between the frame and the fluid. The function of the multiplier pair is weak enforcement of kinematic continuity. This configuration completely decouples the structure and fluid models because each "talks" to the frame through node-collocated multipliers, and not directly to each other. Decoupling simplifies the construction of separate discretizations using different mesh generation programs, the use of customized solution methods (in particular methods available in existing codes) and the implementation of parallel processing.

To advance the solution in time, the LLM interface treatment is combined with a partitioned solution procedure. The time-stepping computations are organized in a way that *eliminates the conventional local prediction step* characteristic of staggered solution procedures. The next-step interface variables, Lagrange multipliers and frame accelerations, are obtained by solving an algebraic system of equations. Interface forces are fed to advance the fluid and structure state. The implicit interface treatment forestalls the well known stability degradation caused by conventional prediction schemes while retaining the desirable localization features of partitioned analysis procedures. Numerical computations indicate that if A-stable integration schemes, such as the trapezoidal rule, are chosen for the fluid and structure, the coupled system retains A-stability, and thus the time step is controlled by accuracy only. This result is proven in Appendix A for the Newark time integrator under certain restrictions. The use of implicit–explicit integration schemes and subcycling remains to be investigated.

2. Driver application problem

The driver application for testing this coupling method on fluid–structure interaction (FSI) problems is a concrete dam on flexible soil. The dam is subject to seismic excitation through base ground motion. Fig. 1 depicts a cross section of a realistic problem of this nature, in which abutments are not shown. (This is not an actual dam configuration, but a composite pieced together from several

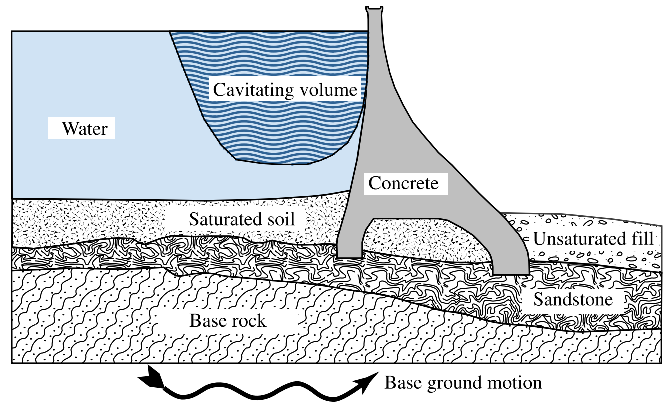

Fig. 1. Concrete dam on a flexible foundation subjected to seismic excitation.

construction and site scenarios. Calculations reported in a sequel paper [46] were carried out on 2D and 3D models of existing dams.)

Model-based simulations involve the interaction of the structure, near-field soil and stored water. For seismic response analysis, the water may be modeled as an acoustic fluid since no significant flow develops during the time span of interest. Two ancillary phenomena may occur. First, water near the vibrating dam may be subject to inertial cavitation [10]. This is a highly nonlinear phenomenon caused by a dynamic pressure drop that overcomes the hydrostatic pressure. Over the cavitating volume the macroscopic fluid elastic modulus drops to near zero while the mass density remains sensibly constant. Repressurization produces potentially damaging closure shocks. Second, the reservoir free surface may develop sloshing (gravity wave) oscillations [16,29]. Although sloshing is included in our fluid model, it normally has no significant FSI effect given its localized character and low associated vibration frequencies.

Following standard techniques of partitioned analysis [15,17,34,35] the problem can be divided into three partitions: structure, soil and fluid, as illustrated in Fig. 2a. The structure and soil are treated with standard finite element discretization procedures of structural mechanics, which lead to a system of semidiscrete equations of motion in the nodal displacements. For the acoustic fluid, however, the displacement potential (a scalar field) is the preferred primary variable on account of advantages discussed later. Linking displacement potentials to actual displacements is not a simple matter, since it requires consideration of fluid element patches. For this reason the fluid model is initially formulated in terms of displacements.

In previous FSI work that focused on underwater shock on submarines [15] and shallow depth attacks on surface ships [47], matching was done by transforming fluid pressures to structural node forces and structural velocities to fluid node forces. These are relayed from fluid to structure and vice-versa, at each time step of a staggered solution procedure. Such procedures necessarily incorporate predictors and have to be carefully designed to avoid stability degrada-