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An eXtended Finite Element Method/Lagrange multiplier based approach for fluid–structure interaction

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Abstract

This paper presents a new fixed grid fluid–structure interaction scheme that can be applied to the interaction of most general structures with incompressible flow. It is based on an eXtended Finite Element Method (XFEM) based strategy. The extended Eulerian fluid field and the Lagrangian structural field are partitioned and iteratively coupled using Lagrange multiplier techniques for non-matching grids. The approach allows the simulation of the interaction of thin and bulky structures exhibiting large deformations. Finally, qualitative examples and a benchmark computation demonstrate key features and accuracy of the method.

Keywords: Fluid–structure interaction; eXtended Finite Element Method; Fixed grid; Domain decomposition; Mortar method; Lagrange multiplier

1. Introduction

Fluid–structure interaction is of great relevance in many fields of engineering as well is in the applied sciences. Hence the development and application of respective simulation approaches has gained great attention over the past decades. Some current endeavors in this field are: the advancement from special purpose or special problem to quite general approaches; the desire to even capture very general and complex systems; and the exigent need of robust high quality approaches even for such complex cases, *i.e.* approaches that have the potential to turn over from being a challenging and fascinating research topic to a development tool with real predictive capabilities [43]. Often, when interaction effects are essential this comes along with large structural deformations. However, many available approaches (both in research as well as in commercial codes) lack robustness especially in this situation.

A sketch of the general problem of the interaction of a flow field and a flexible structure is shown in Fig. 1. The interface Γ^{i} separates the structural domain Ω^{s} from the fluid domain Ω^{f}. Most research and commercial codes that are available for simulations of the interaction of flows and flexible, often thin-walled, structures are based on the Arbitrary Lagrangian Eulerian (ALE) method. These approaches go back to early works like [3,4,12,20,22,33]. The essential feature of ALE based methods is that the fluid field is formulated and solved on a deforming grid. This grid deforms with the structure at the interface and then the grid deformation is extended into the fluid field.

But even the most advanced and best cultured ALE based scheme once comes to its limits where only re-meshing helps. At the latest in such situations, one might be tempted to turn over to approaches that work with a fixed grid. Here, the interface is described either explicitly, using some kind of Lagrangian interface markers or a Lagrangian structural discretization, or implicitly, using *e.g.* level-set functions on a fixed fluid grid. Changing properties and discontinuities in the fluid solution have to be taken care of with modifications on the fluid equations and/or fluid discretization.

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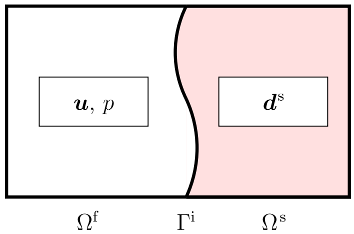

Fig. 1. Problem setup: fluid field Ω^{f}, structural field Ω^{s} and the conjoined interface Γ^{i}.

Prominent fixed grid methods for incompressible flow include the Immersed Boundary (IB) method [35,36] and its many derivations [24,26,28]. It is capable to simulate thin and deformable boundaries and fully fledged, deformable 3d structures submersed in incompressible flow [45,48]. An approach with many similarities to the IB method is the so called Distributed Lagrange Multiplier/Fictitious Domain (DLM/FD) method [17,18]. Originally, the approach was developed for rigid particles with translational and rotational degrees of freedom. The DLM/FD methods have since been extended to simulate thin, deformable structural surfaces [1,11,40] as well as to flexible and fully fledged structures [47]. Both methods have in common a Lagrangian mesh for the structure moving on top of the fluid mesh and forcing the fluid material inside the structure to deform as the structure. Other fixed grid or Cartesian grid methods are, among others, Fedkiw et al. [13], Cirak and Radovitzky [9] and Löhner et al. [27].

One very attractive feature of ALE methods is that fluid and structure domain can be discretized each on its own such that one only has to deal with the coupling of fitting or non-fitting surface meshes. However, when treating a structure on a fixed-fluid grid, the fluid–structure interface essentially divides the fluid domain in a physical flow field and a fictitious field that may be discretized and solved, but has no physical meaning to the FSI problem.

The problem with most fixed grid methods is that there is no way to decouple physical and fictitious domain properly with respect to the kinematic and stress fields and switch of the fictitious flow field calculation to reduce unnecessary computations. In most DLM/FD and IB approaches to date, if used with volume occupying structures, the coupling takes place between the fictitious flow field and the structure domain, not the surface. Since the structure movement is tied to the fictitious flow, it is forced to deform in an incompressible way and artificial viscosity modifies the real physics of the structural domain. Of course the error made by such coupling might not be prohibitive high in specific situations, however, it still poses additional sources of errors for an already complex physical problem.

In summary we propose a number of requirements that future fixed grid methods have to fulfill to be applicable to general structures interacting with incompressible flow: the physical and fictitious flow region are physically decoupled such that no energy transfer occurs across the interface, the coupling takes place only along the interface, there is no mesh size dependency between fluid and structure mesh and it should be possible to turn off most parts of the fictitious domain in order to reduce memory and performance consumption. Despite of the fluid mesh deformation problem, ALE methods, when properly implemented, could be seen as a reference in terms of applicability, accuracy and numerical stability as well as generality with respect to structural material models.

In an attempt to meet all of these requirements, we propose a partitioned iterative coupling scheme between a standard Lagrangian structural description and an Eulerian formulation for the fluid that uses features of the eXtended Finite Element Method (XFEM) and DLM/FD methods mentioned above. A related monolithic implementation of another XFEM based approach for compressible flows was published in Legay et al. [25]. The XFEM was originally introduced for the simulation of cracks and other discontinuities in structures [2,32] and has been, close to the topic at hand, extended to problems of two-phase flow [8] and Stokes flow/rigid particle interaction [41]. In this paper, we derive a 3-field FSI approach with an intermediate reference field. The 3-field approach greatly increases the flexibility with respect to discretization techniques and code modularity.

With this XFEM based approach for the fluid, in principal all of the mentioned shortcoming can be addressed, most prominently, there is no influence of the fictitious fluid domain anymore and, for a sufficient large fictitious fluid domain, a significant number of unnecessary fluid unknowns can be removed. The interface can properly represent the discontinuities. The issue of mesh size dependency between fluid and solid mesh can be shifted to an issue of choosing the right lagrange multiplier method [6,7], which itself is a vivid research topic in the domain decomposition community.

The paper is structured as follows: The general FSI problem is stated in Section 2, where we also propose our 3-field approach. The introduction of the intermediate interface field as an external reference allows us to derive the XFEM fluid problem and its coupling to the interface in Section 3 separately from the coupling between structure and interface, which is subsequently described in Section 4. The coupling and the overall solution approach is then derived in Section 5. Finally, we provide example computations to illustrate important aspects and features of the proposed algorithm and a first 'benchmark' computation to verify the correct transfer of physical quantities across the fluid–structure interface in Section 6.

2. Statement of coupled fluid–structure problem

A general fluid–structure interaction problem statement consists of the description of fluid and solid fields, appropriate fluid–structure interface conditions at the common interface and conditions for the remaining boundaries, respectively.