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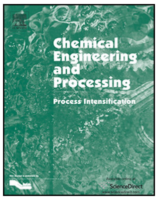

The dissipation rate of turbulent kinetic energy and its relation to pumping power in inline rotor-stator mixers

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ABSTRACT

The theoretical understanding of inline rotor-stator mixer (RSM) efficiency, described in terms of the dissipation rate of turbulent kinetic energy as a function of mixer design and operation, is still poor. As opposed to the correlations for shaft power draw, where a substantial amount of experimental support for the suggested correlations exists, the previously suggested correlations for the dissipation rate of turbulent kinetic energy have not been experimentally validated based on primary hydrodynamic measurements. This study uses energy conservation to reformulate the previously suggested dissipation rate correlations in terms of pumping power which allows for empirical testing. The dimensionless pumping power of three investigated geometrically dissimilar inline RSMs were found to be qualitatively similar to that of centrifugal pumps and decrease linearly with the inline RSM flow number. The previously suggested models for turbulent dissipation in inline RSMs are inconsistent with this observation. Using this reformulation approach, the previously suggested correlation for power-draw is extended to a correlation for dissipation. A new model is suggested based on conservation of energy and angular momentum, and the empiric pumping power relationship. The new model compares well to CFD simulations of total dissiaption and show reasonable agreement to emulsification drop size scaling.

1. Introduction

Rotor-stator mixers (RSMs) are used for mixing and dispersion of multiphase systems in many applications of chemical engineering – e.g. in food, pharmaceutical and cosmetic industry [1,2]. RSMs can be operated in batch, semi batch and inline (continuous) mode of operation, and there is an increased industrial interest in transitioning from batch RSMs to inline mode of operation due to the economic advantage of continuous production.

However, application and design of inline RSMs suffers from a lack of theoretical understanding. Mixing and dispersion in batch RSM are already less theoretically developed than for other high intensity processing equipment such as impeller-systems or high-pressure homogenizers [2,3]. Moreover, inline RSMs offer additional complications compared to batch mode of operation.

The inline RSM can be seen as a hybrid between a centrifugal pump and a batch RSM. The rotor-stator device is typically of the same or similar design as in batch RSM, but is mounted in a narrow casing with inlet and outlet piping. Similar to a centrifugal pump, the inlet pipe is directed towards the central axis of the rotor, the fluid is accelerated radially by the torque of the rotor blades and re-directed towards the peripheral outlet at increased static pressure. As for batch RSMs, high turbulence intensity is generated in or near the holes or slots of the stator screen. Different manufacturers offer different rotor-stator designs, e.g. in terms of number and size of stator holes, design of rotor blades and in the number of rotors and stators.

During the last decade, significant advances have been made in understanding the inline RSM power draw, i.e. the shaft power required to obtain a desired rotor speed at a set volumetric flowrate. Under turbulent conditions, it has been suggested that the shaft power draw consists of three terms [4–8]

$$P_{\text{shaft}} = \Pi_{\text{rot}} + \Pi_{\text{flow}} + \Pi_L = N_{P0}\rho N^3 D^5 + N_{P1}\rho Q N^2 D^2 + \Pi_L \qquad (1)$$

where N_{P0} and N_{P1} are design dependent constants. The first term in Eq. (1), Π_{rot}, is similar to the power draw of a batch RSM and describes the effect of rotor speed. The second term, Π_{flow}, describes the effect of increasing power draw with increasing flowrates and the third, Π_L, is a loss-term describes power lost in

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Notation

Abbreviations

CFD	Computational fluid dynamics
TKE	Turbulent kinetic energy
RSM	Rotor-stator mixer

Roman

A	Rotor blade area, m^2
A_{hole}	Total stator hole area, m^2
A_{stat}	Total stator screen area, m^2
c_1, c_2	Empirical constants in Eq. (18)
D	Rotor diameter, m
h	Rotor blade height, m
L_{jet}	Characteristic length of stator hole exit jet, m
N	Rotor revolution speed, Hz
N_P	Batch RSM power number
N_{P0}, N_{P1}	Inline RSM shaft power-draw constants
N_Q	Inline RSM flow number
P_{diss}	Total turbulent dissipation power, W
P_L	Loss power, W
P_{pump}	Pumping power, W
P_{shaft}	Shaft power draw, W
P'_{shaft}	Loss-corrected power-draw (P_{shaft}–P_L)
$P_{thermal}$	Total increase in fluid thermal energy, W
Q	Volumetric flow rate, m^3 s^{-1}
r_1	Inner rotor blade radius, m
r_2	Outer rotor blade radius, m
T	Rotor blade torque, Nm
U, U'	Rotor blade tip-speeds (see Fig. 3), m s^{-1}
V	Volume, m^3
V_{diss}	Dissipation volume, m^3
V_θ, V'_θ	Angular components of the fluid velocity (see Fig. 3), m s^{-1}

Greek

α	Fraction of dissipation place in the effective region of emulsification
β	Blade angle, rad
δ	Rotor-stator clearance, m
Δp_D, Δp_S	Dynamic and static components of the pressure gradient over the RSM, Pa
ε	Dissipation rate of TKE, m^2 s^{-3}
η_{pump}	Pumping power efficiency
Π_{flow}, Π_{rot}, Π_L	Flow, rotation and loss terms in the shaft power draw expression of Eq. (1), W

bearings or due to vibration [8]. Note that the terms of the shaft power-draw correlation are denoted by Greek symbols (as opposed to the shaft power which is denoted by Latin P). This convention is followed throughout the text and will become important in distinguishing the correlation terms from the power derived from the energy balance in Section 2.

Eq. (1) has obtained substantial experimental support through the last decade [4–9]. More recently, a correction to Eq. (1) has been suggested by Jasinska et al. [10] adding two additional terms. However, as noted by the authors (Ref. [10,p. 47]), this correction would only apply for flowrates that are typically below those used in technical operation. It should also be noted that RSM designs differ between manufacturers and the generalizability of Eq. (1) is still somewhat unclear since most of the experimental work are on Silverson or Tetra Pak RSM designs. Exceptions exists, it has recently been found that Eq. (1) does not comply with the Conti-TDS RSM design produced by Ystral [11].

Mixing and dispersion efficiency are influenced by the dissipation rate of turbulent kinetic energy (TKE) in the most turbulent region [12], which in turn is influence by the total amount of dissipated energy. It should be noted that the dissipation rate of TKE, or the total dissipated energy, is not equal to the power input for an inline mixer. Thus Eq. (1) cannot be used to calculate the energy available for dissipation without adding more assumptions. As opposed to batch RSM, there is still no generally accepted expression for estimating the dissipation rate of TKE in the effective region of mixing and dispersion at different operating and design parameters.

For a batch mixer, the power available for turbulent dissipation, P_{diss}, relates directly to the power-draw [1,13]

$$P_{\text{diss}} \propto P_{\text{shaft}} = N_P \cdot \rho \cdot N^3 \cdot D^5. \qquad (2)$$

with a design dependent constant N_P.

However, the situation is more complicated for inline systems since some of the power input from the shaft is used for pumping the fluid. Experiments have also revealed that RSMs run in batch and inline differ in how they scale with regards to mixing efficiency [14] and emulsification efficiency [15]. This illustrates that the dissipation rate of TKE in inline RSM differs in a substantial way from that in a batch RSM, i.e. P_{diss} is not equal or proportional to P_{shaft} for inline mode of operation. Moreover, this also implies that it is not obvious that the dissipation power is proportional to the rotation-term in the shaft power-draw correlation (Π_{rot}) as is often assumed, e.g. Refs. [6,10]. More investigations are needed to clarify the relation between the terms in the shaft power-draw correlation and the dissiaption.

CFD studies have been conducted with the intention to shed light on the hydrodynamics of inline RSMs [10,16–18]. Especially with micromixing simulations, a good correlation between experimentally observed mixing and simulations based on local dissipation rate of TKE have been obtained [18]. This illustrates that the spatially resolved dissipation rates obtained from CFD are a fruitful starting point for theoretical discussions. In a recent study we have showed that the lower flow through the stator screen in inline RSMs, as compared to batch RSMs, results in a shift of the position where turbulent kinetic energy is dissipated, thus, suggesting a different mixing and breakup mechanism in the inline RSM than in the batch RSM [19].

CFD is useful for detailed investigations. However, it is highly time consuming and is therefore often complemented by empiric correlations, especially in discussing scale-up and interpreting emulsification results. Several suggestions for correlations describing how dissipation rate of TKE scales with operating and design parameters for inline RSMs have been presented [6,10,15,20,21]. However, several of these correlations are contradicting, and no consensus has yet been established even on such basic aspects as scaling with flowrate, rotor speed and diameter. Moreover, experimental studies trying to correlate emulsion drop diameters under different operating conditions to these previously suggested correlations of dissipation rate of TKE, generally conclude that there is poor fit between the two [6,15].

In summary, our current understanding of how mixing and dispersion efficiency depends on operating and design parameters needs to be improved. And an important step in this direction is to describe the scaling of dissipation rate of TKE in the effective region of mixing and dispersion with design and operating parameters. Of special importance is how the dissipation power relates to the terms in the shaft power draw correlation. The specific objective of this contribution is to compare the previously suggested models for dissipation rate of TKE in inline RSMs with special attention to

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