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Assessment of severity of nonlinearity for distributed parameter systems via nonlinearity measures

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ABSTRACT

The performance of controlled distributed parameter systems (DPSs) not only depends on the controller, but also on the dynamic nature of the process itself. One of the primary factors affecting DPS control is process nonlinearity. In many situations, the extent and severity of nonlinearity is the crucial characteristic in deciding whether linear system analysis and controller synthesis methods are adequate. However, due to their spatio-temporal coupling, traditional nonlinearity measures cannot be directly applied to nonlinear DPSs. In this study, a nonlinearity measure method to quantify the severity of nonlinearity for a class of DPSs is proposed. First, time/space separation and model reduction are carried out using proper orthogonal decomposition (POD). Thus, an optimal linear time-invariant model with a low-order is obtained through the optimization of a spatio-temporal error while full state feedback is incorporated in order to stabilize the linear model. Finally, nonlinearity quantification for DPSs is calculated using the obtained stable linear time-invariant system. The complexity of the calculations for nonlinearity measures is greatly reduced after the model reduction using POD. This method easily estimates the extent to which the process behavior deviates from linearity, which aids in determining whether a linear system analysis and controller synthesis methods are adequate. The nonlinearity quantification indicates that DPSs with smaller values are better approximated by a linear model than DPSs with larger values in the target time/space domain. The effectiveness of the proposed method is illustrated using two numerical examples.

1. Introduction

Advanced industrial technological fields, including semiconductor manufacturing, chemical engineering, and materials engineering have a growing demand for control over the flow of fluid, temperature fields, and product size distributions [1]. Considering that their input, output, state, and parameters vary both temporally and spatially, these physical and chemical processes are known as distributed parameter systems (DPSs) [2–9]. In recent years, a number of research groups have focused on issues relating to model reduction and control of DPSs, including Duan [10,11], Park [12,13], Li [14,15], Christofides [16,17], Xie [18,19] and others [20–23]. However, it has become evident that the performance of the controlled processes not only depends on the controller but also on the dynamic nature of the process itself. As demonstrated by Ziegler and Nichols [24], a poor controller is often capable of an acceptable performance on a well-designed process, whereas the finest controller may not deliver the desired performance on a poorly designed process. In regards to process design, Lu [25,27] began to focus on the simplification of the nonlinearity and complexity of a system. An optimally designed process will have satisfactory dynamic characters under a suitably designed parameter. Thus, the nonlinearity and complexity of process dynamics needs to be considered and assessed in order to optimize the control of DPSs.

The effect of process nonlinearity on input/output behaviors is one of the most relevant factors underlying DPS control problems. In control engineering, many nonlinear control techniques have been developed in order to control nonlinear DPSs. However, these techniques must satisfy many stringent conditions [26], which often results in them being impracticable. Alternatively, many nonlinear DPSs are often approximated by a linear model since they can be described very well by this method in the time/space domain of interest. As a result, many linear control methods can be easily used to control nonlinear DPSs [27]. When using these alternative approaches, the extent and severity of the inherent nonlinearity of a system is the crucial characteristic when deciding whether a linear system analysis and controller synthesis methods are adequate

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[28]. Recently, many studies have contributed to the nonlinearity measurement, which is used to estimate the extent to which the process behavior deviates from linearity [29].

The fundamental idea underlying all nonlinearity measures is to compare the properties of the existing nonlinear system with another linear system in an appropriate norm. Nonlinearity measures were first proposed by Desoer and Wang [30] in order to quantify the degree of nonlinearity in the input-output mapping of a system. Nikolaou [31,32] and colleagues evaluated the difference between the best linear model for different inputs and nonlinear systems. Allgower [33,34] also performed an evaluation of the poorest input signals, which can be computed using functional expansions [35]. Guay et al. [36,37] quantified both steady and dynamic nonlinearities using curvature matrices. Sun and Hoo [26] measured the nonlinearity of single input-single output (SISO) systems by calculating the upper and lower linear boundaries of nonlinear systems. Hahn and Edgar [38] introduced a gramian-based approach to nonlinearity quantification in order to measure the nonlinearity of a control-affine process over a given operating region. However, until now, there have been few reports of applications of nonlinearity measures to DPSs; this is likely due to the fact that traditional nonlinearity measures do not have spatio-temporal natures. Although the methods described above have numerous applications for nonlinearity quantification of dynamical systems, they are restricted to the temporal domain.

The main objective of this work was to develop a nonlinearity quantification method for a class of nonlinear DPSs that provides both the magnitude of the spatio-temporal modeling error and an optimal linear model. This stable linear model can be used to design a relatively simple and easy-realization controller in the presence of a nonlinear modeling error. Nonlinearity quantification calculations involve several steps. First, model reduction for a nonlinear DPS is carried out using POD, while empirical eigenfunctions (EEFs) and a low-dimensional nonlinear ODE system can be derived simultaneously. Using specified random input signals, the best linear model is then obtained using the PSO method for a spatio-temporal error while full state feedback is incorporated during the optimization process in order to stabilize the linear time-invariant model. Finally, the stable linear time-invariant system obtained is used to calculate the value of nonlinearity measures for DPSs. The computation complexity is reduced using POD for model reduction, and the nonlinearity quantification indicates that DPSs with smaller values are better approximated by a linear model than DPSs with larger values in the target time/space domain. The effectiveness of the proposed method is illustrated in the two following numerical examples. The results show that the two DPSs have different degrees of nonlinearity in the target time/space domain.

2. The definition of nonlinearity quantification

The assumption is made that a class of nonlinear DPSs is governed by a partial differential equation (PDE) with following state description:

$$\frac{\partial X(z,t)}{\partial t} = \frac{\partial}{\partial z}\left(D(X(z,t))\frac{\partial X(z,t)}{\partial z}\right) - \nu(X(z,t))\frac{\partial X(z,t)}{\partial z} + F(X(z,t)) + U(z,t) \quad (1)$$

Eq. (1) is considered on a bounded spatial domain Ω and is subject to a number of boundaries and initial conditions. $t \in [0, \infty)$ is the time variable, $z \in \Omega = [0, M]$ is the spatial coordinate, and only one spatial-dimension is considered. $X(z, t)$ is the vector of the state variable and $D(X(z, t))$ and $\nu(X(z, t))$ are functions of $X(z, t)$.$U(z,t) = \sum_{i=1}^{p} u_i(t)h_i(z)$ denotes the vector of the manipulated spatio-temporal input, where $u_i(t)$ and $h_i(z)$ denote the temporal input and corresponding spatial distribution, respectively. $F(X(z, t))$ is a nonlinear function containing spatial derivatives for $X(z, t)$.

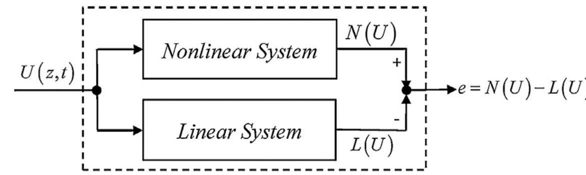

Fig. 1. Setup for the comparison of a nonlinear system and a linear system.

Nonlinearity measures represent an approach to systematically quantify the extent of the nonlinearity in a dynamical system. The concept underlying nonlinearity measures is based on the distance between a nonlinear operator and a suitable linear operator. In other words, the fundamental objective of all nonlinearity measures is to compare the properties of the nonlinear system of interest with a given linear system in an appropriate setup. The most common setup is depicted in Fig. 1.

The signals $U(z, t)$, $N(U)$ and $L(U)$ represent the spatio-temporal input and the trajectories of the nonlinear and linear systems, respectively. Without a loss of generality, it is assumed that the operating points of two systems are both 0. The error signal is set as the difference between that of the nonlinear system and the trajectory of the linear system, which contains the information regarding how well the nonlinear system is approximated by the linear model. In order to quantify this error, a norm of the spatio-temporal domain is defined as follows:

$$\|X(z,t)\| = \sqrt{\int_{\Omega}\int_{0}^{\infty} |X(z,t)|^2 dt dz} \quad (2)$$

The following definition of the nonlinear measure of Eq. (1) for quantification is used:

$$\delta_N = \inf_{L \in \Theta} \sup_{\substack{U \in \Gamma \\ \|N(U)\| \neq 0}} \frac{\|N(U) - L(U)\|}{\|N(U)\|} \quad (3)$$

δ_N represents the norm of the error of trajectories when the worst case input $U \in \Gamma$ is considered to excite the nonlinear DPSs, while Γ is the set of input signals. The best approximation of $L(U)$ is chosen among the set of all causal stable linear systems Θ, such that the resulting worst case error is minimized, and Θ denotes the set of all of the linear systems. The notion of nonlinearity measures was first proposed for the ordinary differential equation (ODE) system [30], in which nonlinearity quantification was defined for nonlinear dynamical systems.

Multiple types of signals with given spatial distributions in Γ can be used as the spatio-temporal inputs for the DPSs (1). In this paper, the random temporal inputs with given spatial distributions are considered to excite the DPSs, and nonlinearity quantification can be computed under this input condition. This results in a simpler nonlinearity measure definition, which may be more suitable in numerical calculations.

$$\delta_N = \inf_{L \in \Theta} \frac{\|N(U) - L(U)\|}{\|N(U)\|} \quad (4)$$

The δ_N represents the "relative error" of the output of the linear model $L(U)$ that best approximates the nonlinear system $N(U)$. If $\delta_N = 0$, then the behaviors of a nonlinear system $N(U)$ can be reproduced by a linear system $L(U)$ for the given input U. Conversely, if $N(U)$ is linear for the given input U, then the best linear approximation is $N(U) = L(U)$, and the nonlinearity measure becomes zero. On the other hand, it is always possible to use a linear approximation for the linear system that yields a zero output (zero operator)