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On the description of localization and failure phenomena by the boundary element method

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Abstract

The application of the boundary element formulation for non-linear structural problems involving localization and failure phenomena is discussed in this paper. In order to overcome the well-known mesh dependency observed for local continua, a non-local damage model is used. An implicit boundary element formulation is proposed and the underlying consistent tangent operator defined. The formulation is based on the classical displacement and strain integral representations and allows the description of snap-back phenomena. Numerical examples are provided to illustrate its capabilities.

Keywords: Boundary element; Nonlocal damage; Arc-length technique; Instabilities; Localization

1. Introduction

Conventional constitutive relations do not include any internal length scale and therefore do not predict any size effect. Many observed phenomena are known in the literature whereby the smaller is the size, the stronger is the response. Indentation hardness of metals and ceramics increases as the size of the indenter is decreased [1]. The strengthening of metals by a given volume fraction of hard particles is greater for small particles than for large ones, for the same volume fraction of reinforcement [2]. Fine grained metals are stronger than those with coarse grains [3]. Finally, torsion experiments have been presented in [4] which show a definite size effect in the range of very thin copper wires.

Localization phenomena are extensively analysed in the context of the Finite Element Method (FEM) in an attempt to improve the numerical simulation of structural failures. There has been however only limited interest in this context with the Boundary Element Method (BEM). In [5,6] application of the BEM to a class of non-local damage models was considered. In [7,8], applications of the Boundary Element Method (BEM) to non-linear problems where localization phenomena take place were considered. In [7,8] gradient plasticity was used in order to avoid difficulties related to local continua while in [9,10] the integral concept has been used to regularize damage effects. In [11] the authors show the first attempt to apply arclength techniques in the BEM but only for local continua.

The presence of strain softening (and/or non-associativity) in the constitutive behaviour brings great difficulties to classical (local) continuum theories in the description of localization phenomena. The associated boundary value-problem is no longer mathematically well posed (see [15]) after the onset of localization and local continua allow for an infinitely small

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bandwidth in shear or in front of a crack tip [14]. At the numerical level, these difficulties translate into the well-known mesh dependence of solutions. Different approaches have been proposed to overcome these difficulties. The idea is to enrich the continuum with non-conventional constitutive relations in such a way that an internal or characteristic length scale is introduced. Examples of such theories are the Cosserat continua [20], the gradient theories [19,17] and the non-local models [18]. This paper uses these non-local theories in the framework of Continuous Damage Mechanics to describe failure in structural members with the Boundary Element Method. Another feature of this work is the consideration of snap-back phenomena that are present in the class of problems studied here. Though the formulations can be used in a more general setting, for conciseness of the presentation, only a scalar isotropic damage model is adopted. An implicite boundary formulation is developed with the consistent tangent operator defined and used to solve the non-linear algebraic system. This is done in the light of the procedure introduced into the boundary element formulation by Bonnet and Mukherjee [22] and Poon et al. [23], using a scheme similar to the one proposed by Simo and Taylor [24] for finite elements.

The paper is structured as follows: Section 2 summarizes the constitutive equations that we have in mind in their local and non-local formats. For simplicity, a simple isotropic damage model is considered. Extension to more involved damage models poses no difficulties. In Section 3 we recall the integral representation for inelastic structures. This integral representation is discretized in time in Section 4 and allows to obtain the finite step problem, i.e. the boundary value problem to be solved over a time step. This solution is carried out in Section 5 by a boundary element formulation. It is presented there in its classical form leaving the consideration of localization and snap-back phenomena to Section 6. These phenomena are controlled by a generalized arc-length procedure. First and simple examples and numerical simulations are provided in the last section to show the abilities of the numerical techniques that have been developed so far to deal with problems involving localization, snap-back and failure phenomena.

2. Constitutive and field equations for damage problems

2.1. Local formulation

Continuum Damage Mechanics (CDM) deals with the load carrying capacity of solids without major cracks but where the material itself is damaged due to the presence of microscopic defects such as microcracks or micro-voids. For a general and detailed presentation of CDM, the reader is sent to the book of Lemaitre and Chaboche [12,13]. In this paper and for sake of clarity we focus on simple elastic-damage formulations although most of the developments may be extended to more elaborate models including plasticity and other phenomena. In the context of continuous thermodynamics, the behaviour of such material may be described by a scalar-valued internal damage variable D defining the internal state of the material. The parameter D reflects the amount of damage which the material has experienced. It starts at zero (sound material) and grows to one (fully damaged material corresponding to complete loss of coherence). The corresponding constitutive relations derive from the Helmoltz free energy Ψ

$$\rho\Psi = \rho\Psi(\epsilon_{ij}, D) = \frac{1}{2}(1-D)E_{ijkl}\epsilon_{ij}\epsilon_{kl} \tag{1}$$

depending on the strain tensor ϵ_{ij} and the damage variable D. ρ is the mass density and E_{ijkl} is the forth-order tensor of the elastic sound material. The stress tensor is given by

$$\sigma_{ij} = \rho\frac{\partial\Psi}{\partial\epsilon_{ij}} = (1-D)E_{ijkl}\epsilon_{kl} \tag{2}$$

and the thermodynamical force Y conjugated to the damage variable D is

$$Y = -\rho\frac{\partial\Psi}{\partial D} = \frac{1}{2}E_{ijkl}\epsilon_{ij}\epsilon_{kl}. \tag{3}$$

The quantity Y is also referred to as the damage strain energy density release rate. Initiation and growth of damage is formulated through a damage criterion involving the loading function $f(Y;D)$ and the evolution law

$$\dot{D} = \dot{\lambda}\frac{\partial f}{\partial Y}(Y;D), \tag{4}$$

where the multiplier $\dot{\lambda}$ in Eq. (4) satisfies the classical (loading/unloading) Kuhn–Tucker conditions

$$\dot{\lambda} \geqslant 0, \quad f \leqslant 0, \quad \dot{\lambda}f = 0 \tag{5}$$

and is calculated (when it is positive) from the consistency condition

$$\dot{f}(Y;D) = \frac{\partial f}{\partial Y}\dot{Y} + \frac{\partial f}{\partial D}\dot{D} = 0. \tag{6}$$