Collision of Two Light Quanta

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The recombination of free electrons and free positrons and its connection with the Compton effect have been treated by Dirac before the experimental discovery of the positron. In the present note are given analogous calculations for the production of positron electron pairs as a result of the collision of two light quanta. The angular distribution of the ejected pairs is calculated for different polarizations, and formulas are given for the angular distribution of photons due to recombination. The results are applied to the collision of high energy photons of cosmic radiation with the temperature radiation of interstellar space. The effect on the absorption of such quanta is found to be negligibly small.

TWO simultaneously acting light waves with vector potentials

$$\mathbf{A}_j=\mathbf{a}_j^* \exp \{-(\omega_j t-\mathbf{k}_j\mathbf{r})\} + \mathbf{a}_j \exp \{i(\omega_j t-\mathbf{k}_j\mathbf{r})\} \quad (1)$$

are considered as acting on an electron. Under the influence of the waves a single electron wave function $\psi^{(0)}$ is changed, and the perturbed function may be expanded according to powers of a, a^*. The phenomena of pair production and of recombination have to do with the terms in $a_1^*a_2^*$ and a_1a_2, respectively, as is obvious from the theory of quantization of light waves. We consider first the pair production. We let the function $\psi^{(0)}$ represent an electron in a negative energy state. It is convenient for practical applications to normalize $\psi^{(0)}$ so as to have the electron density equal to unity. It is also unnecessary to use quantized light waves in the pair production problem, since the results with quantized waves are known to be identical with those obtained by means of ordinary waves. As a result of the calculation one finds that at a time t after the application of the waves the wave function contains a term which may be interpreted as referring to an electron in a positive energy state with a momentum and a spin coordinate which are functions of the original momentum and spin and of the momenta and polarizations of the light quanta. The density of electrons corresponding to this wave function may be put into the form

$$(e^2/mc^2)^2|a_1|^2|a_2|^2B|1-\exp(-it\delta W/\hbar)|^2/(\delta W)^2. \quad (2)$$

Here B is a dimensionless number depending on initial momenta and spin and the polarizations of the quanta. δW is the difference in energy of the initial and the final states. Thus if $W=-|W|$ is the energy of the electron in its initial state and if $h\nu_1$, $h\nu_2$ are the energies of the quanta, then

$$\delta W=c(p_2^2+m^2c^2)^{\frac{1}{2}}+W_1-h\nu_1-h\nu_2; \qquad W_1=-W, \quad (3)$$

where $$\mathbf{p}_2=\mathbf{p}_1+\mathbf{P}_1+\mathbf{P}_2 \quad (4)$$

is the final momentum of the electron and $\mathbf{P}_1$, $\mathbf{P}_2$ are the momenta of the quanta. The total electron density due to the two light quanta is obtained by summing expression (2) over all possible states of negative energy. The equal and opposite spin directions for every $\mathbf{p}_1$ contribute to the density. One is thus only interested in the average for B over the different directions σ of the positron spin. This average will be called $\bar{B}^\sigma$. There are $2p_1^2dp_1d\omega_1\cdot V/h^3$ electronic states of negative energy in the fundamental volume V for which the momentum is $\mathbf{p}_1$ and the direction is within the solid angle $d\omega_1$. Each of these has a density $1/V$. The number of positron electron pairs produced per cm^3 corresponding to the absolute value of positron momentum being between p_1 and p_1+dp_1 in the direction $-\mathbf{p}_1$ and in solid angle $d\omega_1$ is thus obtained from (2) by multiplying it by $2p_1^2dp_1d\omega_1/h^3$. Integrating over dp_1, and making use of

$$d(\delta W)=c^2[p_1/W_1+\mathbf{p}_1\mathbf{p}_2/p_1W_2]dp_1, \quad (5)$$

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which follows from (3) and (4), and substituting

$$|a|^2=(c/2\pi\nu^2)I=(hc/2\pi\nu)N, \qquad (6)$$

where I is the intensity of each beam and N is the number of light quanta per cm^2 per second, one obtains the probability of pair formation per unit solid angle of the positron and per unit volume of the space in which the light quanta collide, as

$$\frac{2p_1^2(e^2/mc^2)^2N_1N_2\bar{B}^\sigma t}{h\nu_1h\nu_2[p_1/W_1+\mathbf{p}_1\mathbf{p}_2/p_1W_2]}. \qquad (7)$$

Here $(-\mathbf{p}_1)$, $\mathbf{p}_2$ are the vectors representing the momenta of the positron and electron, p_1, p_2 are their absolute values and W_1, W_2 are, respectively, the energies of the positron and electron. It is convenient to express the above probability in terms of an effective collision area σ. A convenient definition is to express (7) as $tN_1N_2\sigma/c$. The effective collision area σ thus defined corresponds to a picture of N_1 light quanta per unit area per second traveling through space in which the quanta $h\nu_2$ are thought of as being distributed with their density N_2/c. This definition is arbitrary but convenient for transformations to other frames of reference. It should be noted, however, that if two beams of quanta are shot against each other head-on then the number of pairs produced is

$$A(\sigma/2)\int N_1dt\cdot\int N_2dt, \qquad (8)$$

where A is the common cross-section area of the two beams, the factor 1/2 arising from the fact that both beams travel against each other with the velocity of light. The effective collision area for head-on collisions when expressed in terms of the numbers of quanta shot at each other rather than in terms of the density of one of them is thus $\sigma/2$. We have

$$\sigma=\frac{2cp_1^2(e^2/mc^2)^2\bar{B}^\sigma}{h\nu_1h\nu_2[p_1/W_1+\mathbf{p}_1\mathbf{p}_2/p_1W_2]}. \qquad (9)$$

The recombination probability of electrons and positrons may also be expressed in terms of the quantity B used in (2). One starts with an electron being in a positive energy state and considers the terms in a_1a_2 according to the method of quantizing light waves. The probability for the electron to jump into the hole may then be calculated for this pair of light waves. Using electron and positron states normalized to 1, i.e., of density $1/V$, a result is obtained which is similar to Eq. (2). It differs from (2) only through the replacement of every $|a|^2$ by the initial expectation of a^+a. Thus the contribution to the emission probability due to the possible cooperation of a pair of light waves is obtained by substituting

$$|a|^2=(c/2\pi\nu^2)I$$

and then letting $\quad I=h\nu c/V. \qquad (10)$

Consider a positron with a definite spin and momentum going through an electron distribution also having a definite spin and momentum. In order to obtain the emission probability we must sum expression (2) modified in accordance with (10) over all possible pairs of light quanta. Thus for given momenta of the light quanta 1, 2 each light quantum can have two perpendicular and independent directions of polarization s_1, s_1' and s_2, s_2', respectively, and one obtains contributions to the emission probability due to every possible combination of polarizations. Thus one has the emission probability as a sum of terms

$$(h^2c^4/4\pi^2\nu_1\nu_2)(e^2/mc^2)^2V^{-2}\beta(1,2)$$
$$\times|1-\exp(-it\delta W/\hbar|^2/(\delta W)^2, \qquad (11)$$

where $\beta(1, 2)$ should be made to take in turn the values $\beta(s_1, s_2)$, $\beta(s_1, s_2')$, $\beta(s_1', s_2)$, $\beta(s_1', s_2')$. If it happens that the pair production calculation for quanta 1, 2 gives for the assigned positron spin also the assigned electron spin then $\beta(1, 2)=B(1, 2)$. In general, however,

$$\beta(1,2)=\frac{|(\psi_\sigma^*,\psi_{\sigma'})|^2}{(\psi_{\sigma'}^*\psi_{\sigma'})(\psi_\sigma^*\psi_\sigma)}B(1,2), \qquad (12)$$

where ψ_σ, $\psi_{\sigma'}$ are, respectively, the electron state with specified spin and the electron state which arises from the positron state under the action of the light quanta 1, 2. The probability of having recombination in which light quantum 1 has the polarization s_1 while the polarization of light quantum 2 is subject to no restriction is

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