Journal of Process Control 47 (2016) 111–120

Contents lists available at ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

4-Degree-of-freedom anti-windup scheme for plants with actuator saturation☆

Li Zhong-Shen [a], Mo Xiao-Qin [b], Guo Shang-Jia [c], Lan Wei-Yao [d], Huang Chun-Qing [d,*]

[a] College of Mechanical Engineering and Automation, Huaqiao University, Xiamen, Fujian 361021, China
[b] Polytechnic Institute, Sanya University, Sanya, Hainan 572022, China
[c] Haikou Power Supply Bureau, Hainan Power Grid Company, Haikou, Hainan 570100, China
[d] Department of Automation, Xiamen University, Xiamen, Fujian 361005, China

ARTICLE INFO

Article history:
Received 2 August 2015
Received in revised form 6 July 2016
Accepted 21 July 2016

Keywords:
Actuator saturation
Anti-windup
IQCs
Stability

ABSTRACT

In this paper, a 4-degree-of-freedom (DoF) anti-windup scheme is developed for anti-windup compensation of plants with actuator saturation. Instead of LMIs (Linear Matrix Inequalities) in modern anti-windup technique, sufficient stability conditions of the proposed anti-windup scheme are derived under the framework of IQC (Integral Quadratic Constraint). The sufficient conditions are formulated as a checking on the positive definiteness of the related transfer function at all frequencies for MIMO systems (or positive sign for SISO systems), which provides a fairly straightforward tuning rule in the frequency domain. Moreover, by virtue of the proposed scheme, control design for disturbance response is decoupled from set-point response. The proposed scheme can be implemented without system states or state estimates. Compared with IMC-based, low-order or dynamic compensation anti-windup schemes, numerical example shows remarkable performance improvement of the proposed scheme for the saturated systems.

1. Introduction

Due to the minimum and maximum limits of physical actuators [13], actuator saturation is common in practice. When the effect of saturation is not taken into account for controller design, it might result in performance deteriorated and even instability of the resulting closed-loop system [23,45]. Control design under actuator saturation has been widely studied in literatures [2,4,10,41]. Roughly speaking, there are two approaches for dealing with actuator saturation [6,13,35]. One approach, which is referred to as the one-step approach, is to design linear (or non-linear possibly) controller directly that attempts to meet all nominal performance specification and handles actuator's constraints simultaneously. Although this approach is satisfactory in principle, it has often been criticized because of its conservatism as well as lack of intuition and lack of applicability to some practical problems [19,31]. Another approach is the two-step approach, which first designs linear controller to achieve nominal performance without considering actuator's saturation, and then employs an anti-windup compensator to minimize adverse effects of saturation on closed-loop performance [6,14,35]. It is only when saturation is encountered that the anti-windup compensator becomes active to modify the closed-loop's behavior to counteract the negative effect of saturation. The two-step approach becomes a popular choice for practical engineers because no restriction is placed upon the nominal linear controller design and it can be retrofitted to the existing controllers which may function very well except during saturation [7,40,43].

In principle, the existing techniques for analysis and synthesis of nonlinear systems, such as the small gain theorem and absolute stability tools like the Circle and Popov Criterion, can be used to design anti-windup compensator for stable linear saturated systems (single-loop systems especially) (see Refs. [7–9,15,17,24,39]). But the design using the above techniques lacks some practicality and hence is somewhat arduous until LMI technique emerges [35]. One of the first applications of LMI is given for anti-windup synthesis in [21] by means of the absolute stability theory involving common Lyapunov functions, but it is not wholly LMI-based since inequalities are bilinear matrix inequalities (which can be linearized by fixing one of the free variables) [28]. Basing on the general framework as presented in [16], Mulder et al. [24] present a wholly LMI-based synthesis method for static anti-windup compensator with a constraint on the L_2 gain from the exogenous input to the regulated output, and reduce the compensator design to a convex optimization problem over LMIs for the first time. Note that the anti-windup framework presented by Weston et al. [43] involves many anti-windup schemes (such as IMC, Hanus

☆ The paper is partially supported by the National Nature Science Foundation of China (61374035).
* Corresponding author.
E-mail address: chqhuang@xmu.edu.cn (C.-Q. Huang).

http://dx.doi.org/10.1016/j.jprocont.2016.07.004
0959-1524/

conditioning, unified conditioning) as special cases, in which the resulting closed-loop system is transformed into an equivalent presentation that is paralleled connected by a intended linear loop and a nonlinear loop cascaded with disturbance filter, and hence bring some convenience for analysis and synthesis of anti-windup compensator [11,29,43]. Using such equivalent presentation, Turner et al. [40] propose a performance map which allows deviation from linear performance to be minimized explicitly via LMIs and then accomplish the synthesis of static and low-order anti-windup compensator with superior performance (in terms of their L_2 gain) obtained. In Refs. [7,9], the design of fixed-order anti-windup compensator with a desired level of L_2 performance is cast as a non-convex optimization problem. It becomes convex in the special cases of static and plant-order anti-windup compensation and hence can be reduced to a standard LMI feasibility problem. Since static anti-windup compensators are most desirable in practice [32,40], a LMI-based design method of static anti-windup compensator is proposed in Ref. [32], the non-convex problem is reduced to an equivalent convex problem by using an appropriate sector transformation. Although the implementation complexity of reduced order compensator is increased comparing with that of the static compensator, reduced-order compensator may work well in some cases and first-order filters is proposed to apply as reduced-order anti-windup compensator [40,20]. Using an iterative convex relaxation of non-convex rank constraint involved, Galeani et al. [5] provide a constructive algorithm for reduced order compensator design, in which a minimal order compensator can be determined and becomes quite useful within the general context of the anti-windup approaches.

The advantage of LMI-based compensation technique is that the compensator design task can be accomplished via convex optimization with different optimal synthesis criteria (such as L_2-gain attenuation or enlargement of the basin of attraction) [36]. LMI-based anti-windup techniques with full order, reduced order and static compensator aforementioned have been successfully applied in practical control systems [26] such as hard disk [12], aircrafts [1], motor speed control [27,20] and missile auto-pilot [5] etc. However, as far as industrial process is concerned, LMI-based anti-windup technique suffers from the following drawbacks: (1) Infeasibility problem is encountered in LMI-based techniques [8,24]. The level of disturbance attenuation can be adjusted to makes LMIs feasible, but it may be not enough to meet the design objective of anti-windup compensation. (2) The demanding of states or state estimates, which is generally required for implementations of dynamic/reduced-order compensators, is undesirable or even impossible to acquire in the practical process. (3) In the framework of LMI-based anti-windup compensation, offset-free set-point tracking and step disturbance rejection, which are usually regarded as design objectives in industrial process, can not be achieved since set-point response along with load disturbance response of the resulting closed-loop system can only be designated as a prior specified level. (4) Some knowledge and understanding based on engineer's experience can not be utilized directly and hence leave little room of intuition for compensation synthesis [5,20,40] since there exist some gap between LMI and engineering understanding of system characteristics.

Motivated by the above drawbacks in LMI-based anti-windup techniques, this paper is to explore new scheme for anti-windup compensation of plants with actuator saturation. A 4-DoF anti-windup scheme is proposed on the basis of transfer function description of plants, in which states or state estimates of plant are not required and infeasibility problem which arises in LMI-based techniques would not be an issue anymore. Especially, the setpoint response is decoupled from the load disturbance response, and in some sense, both of them can be specified arbitrarily. Hence it makes a remarkable improvement of performance possible in comparison with LMI-based anti-windup techniques.

This paper is structured as follows. Some preliminaries regarding the description of plants with actuator saturation are presented in Section 2. In Section 3, the proposed 4-DoF anti-windup scheme is developed and some properties are discussed. As for stability analysis of the resulting system, sufficient conditions are derived on the basis of IQC framework in Section 4, and hence the tuning rule for stability is obtained accordingly. Some further discussion regarding features of the proposed scheme is presented in Section 5. In Section 6, a numerical example is presented to illustrate the remarkable performance improvement of the proposed anti-windup scheme with the comparison of three existing anti-windup techniques.

2. Preliminaries

Consider $n \times n$ linear-time-invariant (LTI) plant:

$$G(s) = \left\{ \frac{\widehat{B}_{ij}(s)}{\widehat{A}_{ij}(s)} \right\}, \quad i, j = 1, \cdots, n \tag{1}$$

where G(s) is a rational strictly proper transfer function matrix; $\widehat{A}_{ij}(s)$, $\widehat{B}_{ij}(s)$ are real coprime polynomials.

In many cases [6,35,40,43], the whole system with exogenous input $\boldsymbol{w} \in \mathbb{R}^{n_w}$ is presented by state-space realization:

$$\begin{cases} \dot{\boldsymbol{x}}_p = A_p x_p + B_p \mu + B_{pw} \boldsymbol{w} \\ \boldsymbol{y} = C_p x_p + D_p \mu + D_{pw} \boldsymbol{w} \\ z = C_z \boldsymbol{x}_p + D_z \mu + D_{zw} \boldsymbol{w} \end{cases}, \qquad G(s) \sim \begin{bmatrix} A_p & B_p \\ C_p & D_p \end{bmatrix} \tag{2}$$

where $\boldsymbol{x}_p \in \mathbb{R}^{n_p}$ is the plant state, $\boldsymbol{u} \in \mathbb{R}^n$ the control input, $\boldsymbol{z} \in \mathbb{R}^{n_z}$ is the regulated output vector used for performance purposes, $\boldsymbol{y} \in \mathbb{R}^n$ is the plant output available for measurement, and A_p, B_p, C_p, D_p, B_{pw} and D_{pw} are real constant matrices with suitable dimensions. In a real physical system, the control input signal $\boldsymbol{u}_i$ of the ith actuator is always limited. Therefore, $\boldsymbol{u}_i$ is constrained such that

$$\boldsymbol{u}^i_{min} \le \boldsymbol{u}_i \le \boldsymbol{u}^i_{max}, \quad i = 1, \cdots, n. \tag{3}$$

This can be formulated by the following saturation function $sat(\cdot)$

$$\hat{\boldsymbol{u}}_i = sat(\boldsymbol{u}_i) = \begin{cases} \boldsymbol{u}^i_{max} & \text{if} \quad \boldsymbol{u}_i > \boldsymbol{u}^i_{max} \\ \boldsymbol{u}_i & \text{if} \quad \boldsymbol{u}^i_{min} \le \boldsymbol{u}_i \le \boldsymbol{u}^i_{max} \\ \boldsymbol{u}^i_{min} & \text{if} \quad \boldsymbol{u}_i < \boldsymbol{u}^i_{min} \end{cases} \tag{4}$$

where $\hat{\boldsymbol{u}}_i$ is the actual control input signal, and $\boldsymbol{u}^i_{max}$ ($\boldsymbol{u}^i_{min}$) is the maximum (minimum) value of the ith actuator. For some constant δ and any bounded signal $\boldsymbol{\zeta} \in \mathbb{R}^n$, there always exists some nonlinear continuous function $N(\boldsymbol{\zeta}_i)$ ($i = 1, \cdots, n$) such that

$$sat(\zeta) = N(\boldsymbol{\zeta}) \cdot \boldsymbol{\zeta}, \tag{5}$$

$$N(\boldsymbol{\zeta}) = \mathrm{diag}\{N(\boldsymbol{\zeta}_i)\}, \ 0 < \delta \le N(\boldsymbol{\zeta}_i) \le 1, \ i = 1, \cdots, n \tag{6}$$

$$N(\zeta_i) = 1 \ \text{if} \ \boldsymbol{u}^i_{min} \le \boldsymbol{\zeta}_i \le \boldsymbol{u}^i_{max}, \quad i = 1, \cdots, n \tag{7}$$

Hence, the inverse of diagonal matrix $N(\cdot)$ always exists, i.e.,

$$I \le N^{-1}(\cdot) \le \frac{1}{\delta} I, \tag{8}$$

$$N^{-1}(\zeta_i) = 1 \ \text{if} \ \boldsymbol{u}^i_{min} \le \boldsymbol{\zeta}_i \le \boldsymbol{u}^i_{max}, \ i = 1, \cdots, n \tag{9}$$

3. 4-DoF anti-windup scheme

Several 2-DoF control schemes are proposed to overcome the so-called water-bed effect between the setpoint response and the load