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Comput. Methods Appl. Mech. Engrg. 195 (2006) 6046–6058

Computer methods in applied mechanics and engineering

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Prediction of the critical Reynolds number for flow past a circular cylinder

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Received 9 June 2004; received in revised form 15 January 2005; accepted 21 October 2005

Abstract

In this paper we attempt to estimate the value of critical Reynolds number, Re_c, for the first wake instability of the wake associated with the flow past a circular cylinder. Linear stability analysis (LSA) and direct time integration (DTI) of the governing equations for incompressible flows are carried out via a stabilized finite element formulation. The generalized eigenvalue problem resulting from the finite element discretization of the equations from linearized stability analysis is solved using a subspace iteration (SI) method to get the most unstable eigenmode. The results from the two methods are in good agreement. The effect of spatial resolution and location of computational boundaries is investigated. It is found that, for high blockage (ratio of the diameter of cylinder to the lateral width of domain), Re_c first decreases and then increases with increase in blockage. It is also observed that the Strouhal number at Re_c is quite sensitive to the blockage. This might possibly explain the scatter in the data from various researchers in the past.

Keywords: Hopf bifurcation; Circular cylinder; Linear stability analysis; Finite element method; Direct time integration; Blockage

1. Introduction

It is well known that at $Re \sim 50$ the steady flow past a cylinder looses stability. This instability becomes stronger with further increase in Re and eventually leads to von Karman vortex shedding. There have been several efforts towards the characterization and understanding of this phenomenon. The shedding is accompanied with increased base pressure, Reynolds stresses and mean drag coefficient [18,27]. There have been several efforts in the past to estimate the critical Reynolds number (Re_c) by different methods: numerical and experimental. A large scatter in the data reported by various researchers is observed. Some of the difficulties encountered in the laboratory experiments to determine the precise value of Re_c have been highlighted by Coutanceau and Bouard [3,4]. One of them is the effect of blockage, relative size of a cylinder with respect to the experimental apparatus. Another factor is the residual turbulence at the inlet which can excite the flow at Re below Re_c [8].

Computational efforts to determine Re_c are usually one of the two types: Direct time integration (DTI) or linear stability analysis (LSA). DTI corresponds to the time-integration of Navier–Stokes equations. The linear stability analysis involves solving an eigenvalue problem and finding the most unstable mode. While the former technique is extremely demanding on CPU time owing to extremely slow growth rates at the onset of instability, the latter is restricted by the memory requirements for a computation with reasonably fine resolution. The resolution/grid-size and the size of the computational domain coupled with the boundary conditions on the lateral boundaries play an important role in the accuracy of the

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0045-7825/$ - see front matter
doi:10.1016/j.cma.2005.10.009

Table 1
Hopf bifurcation for the uniform flow past a circular cylinder: comparison of the critical parameters

Researcher(s)	Re_c	St_c	Method	Grid points	Domain size $L \times H$
Roshko [17]	40	0.120	Experiments		$H/D = 1250$
Berger [2]	50.00	0.12	Experiments		
Coutanceau and Bouard [3]	34–43	–	Experiments		
Williamson [26]	47.90	0.1220	Experiments		$H/D = 150$
Norberg [14,15]	47.4 (±0.5)	0.1177	Experiments		$H/D = 6250$
Gresho [7]	50.00	0.14	FEM	1825	
Jackson [8]	45.403	0.13626	FEM with II	3056	$20D \times 10D$
Zebib [28]	39–43	0.11–0.13	Eigenvalue anal.		
Ding and Kawahara [5]	46.389	0.12619	FEM AR	9870	$36D \times 16D$
Morzynski and Thiele [13]	46.270	0.13451	FDM with NR	3200	
Morzynski et al. [12]	47.00	0.1320	FEM with SI	15,838	$20D \times 10D$
Present calculation	46.877	0.1168	FEM (2D DTI)	40,480	$100D \times 100D$
Present calculation	47.318	0.1169	FEM with SI	24,840	$100D \times 100D$

L is the length of the domain in streamwise direction while H is its lateral width.

computational results. In flow problems where the eigenvalues encountered are close to each other the sequence of modes cannot be resolved unless the discretization is fine enough. Morzynski et al. [12] have pointed out that the eigenvalue calculation demands more mesh points than the corresponding direct numerical simulation over the same geometry. In a DTI, the mesh points are concentrated in the wake and boundary layer regions for adequate resolution. Typically, these regions are known apriori and form only a fraction of the entire computational domain. However, in an eigenvector field it is impossible to predict the regions associated with large gradients. As a general rule, fine resolution in the wake region and approximately uniform spacing in the rest of the domain is employed.

Table 1 gives the value of Re_c reported by various researchers. For numerical studies, the method used and domain size are also listed. For example, Jackson [8] used the finite element method (FEM) along with Inverse Iteration (II) method and reported $Re_c = 45.40$. Morzynski et al. [12], using FEM and subspace iteration (SI), found $Re_c = 47.00$. Ding and Kawahara [5] report $Re_c = 46.389$ using FEM and Arnoldi Method (AR). Williamson [26], through laboratory experiments, found it to be 47.90. It is clear from the table that there is a large variation in the values of Re_c and St_c as reported by the different researchers in the past. An attempt will be made here to explain this variation in the critical parameters.

In the present study, DTI and LSA are employed to determine the Re_c. Although, both the methods have been utilized by researchers in the past, to the best of the knowledge of these authors, this is the first effort where the two techniques have been used with similar formulations and computational grids. The effect of the domain size and the resolution on Re_c, using LSA, is also studied.

2. Solution method

Several researchers in the past have carried out stability analysis of incompressible flows using the finite element method. For example, Jackson [8] discussed flow past variously shaped bodies using the subspace iteration method. He utilized a biquadratic interpolation for velocity and piecewise linear discontinuous function for pressure. Morzynski et al. [12] used a penalty formulation with the subspace iteration method to study the stability of flow past a circular cylinder with and without a control cylinder. Ding and Kawahara [5] employed a Krylov subspace method to investigate the linear stability of three-dimensional flow past a circular cylinder and lid-driven cavity flow. A mixed finite element method was used with quadratic interpolation for velocity and linear function for pressure. They also presented a fairly detailed review of efforts from other researchers including those who have used finite difference/spectral methods.

In this paper we present a stabilized finite element formulation that allows one to employ equal-order-interpolation functions for velocity and pressure. To the best of our knowledge, all prior efforts have employed unequal orders of interpolation. The SUPG (Streamline-Upwind/Petrov–Galerkin) and PSPG (Pressure-Stabilizing/Petrov–Galerkin) stabilization technique [24] is employed to stabilize the computations against spurious numerical oscillations.

For carrying out the stability analysis, first, steady-state solution is computed by dropping the time dependent term from the flow equations. The steady-state solutions at various Re are obtained by progressively increasing the Re. It becomes increasingly difficult to obtain the steady-state solutions as Re increases. The eigenvalue problem in the LSA is solved via a sub-space iteration procedure [12]. It involves the LU decomposition of the matrices resulting from the finite-element discretization of the flow equations. The solution to the eigenvalue problem resulting from the linear stability analysis is computationally demanding. It has been observed that the convergence of the eigenvalue problem for Re far away from Re_c is rather slow. To reduce the computational time, a shift-invert transformation is employed.