Journal of Process Control 49 (2017) 1–8

Contents lists available at ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

Implications of discretization on dissipativity and economic model predictive control

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ARTICLE INFO

Article history:
Received 9 June 2015
Received in revised form 31 October 2016
Accepted 14 November 2016
Available online 29 November 2016

Keywords:
Economic model predictive control
Dissipativity
Stability
Discretization
Direct collocation

ABSTRACT

Economic model predictive control, where a generic cost is employed as the objective function to be minimized, has recently gained much attention in model predictive control literature. Stability proof of the resulting closed-loop system is often based on strict dissipativity of the system with respect to the objective function. In this paper, starting with a continuous-time setup, we consider the '*discretize then optimize*' approach to solving continuous-time optimal control problems and investigate the effect of the discretization process on the closed-loop system. We show that while the continuous-time system may be strictly dissipative with respect to the objective function, it is possible that the resulting closed-loop system is unstable if the discrete-approximation of the continuous-time optimal control problem is not properly set up. We use a popular example from the economic MPC literature to illustrate our results.
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1. Introduction

Economic model predictive control (e-MPC), a model predictive control (MPC) approach to the optimal control of systems has recently gained much popularity. The main difference between e-MPC and existing MPC approaches is the nature of the objective function being optimized. While conventional MPC approaches employ a positive-definite function that minimizes deviations from desired set-point, the objective function in e-MPC is a generic cost that is related to the economics of the system's operation.

Two major techniques exist in the literature for designing digital controllers for systems: the emulation method and the direct design method [1–3]. In the emulation method, the controller design is done in the continuous-time domain followed by a discretization of the controller. In the direct design methodology, the digital controller is designed directly using a discretized model or a discrete approximation of the system. While emulation methods do exist for MPC, most standard MPC settings use a direct design that involves discretizing both the system's model and objective function.

Of great importance in the context of economic-MPC is the dissipativity of the system with respect to the economic objective as this is one of the conditions on which the stability proof for the closed-loop system is often based. It has been shown [4,5] that strict dissipativity of the system with respect to the economic cost plays a central role in the stability proofs of the closed loop system. It has also been shown in the economic-MPC literature that if a system is dissipative with respect to the economic objective, then static equilibrium operation of the system is optimal [5–7]. It is therefore important to know what happens to the dissipativity property when the continuous-time setup is '*discretized then optimized*' as done in the direct design method. Is it possible to have a dissipative continuous-time setup and the discretized setup not dissipative? Under what circumstances can these happen and how can we avoid them?

These are the questions we raise and attempt to answer in this paper. We show that due to the approximate discretization of the underlying continuous-time optimal control problem, it is possible to lose the system's dissipativity (with respect to the given economic objective) hence, possible loss of stability of the closed-loop system. We also show that the conventional MPC scheme (where the cost function is designed to be positive-definite) is immune to such behaviour.

This paper is structured as follows: Section 2 introduces the problem statement. The effect of approximate discretization on a linear-quadratic optimal control problem is discussed in Section 3. The effect of approximate discretization is extended to direct transcription methods for solving optimal control problems in Section 4 with the focus on direct collocation methods. Section 5 contains a popular example from e-MPC literature while Section 6 concludes the paper.

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http://dx.doi.org/10.1016/j.jprocont.2016.11.002
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2. Preliminaries

In this paper, we consider the continuous-time, finite horizon optimal control problem

$$J_T^*(x) = \min_{\mathbf{u}} J(x, \mathbf{u}) \triangleq \int_0^T l_c(x(t), u(t))\,dt$$
$$\text{subject to} \begin{cases} \dot{x}(t) = f(x(t), u(t)) & \forall t \in [0, T] \\ x(t) \in \mathbb{X}, u(t) \in \mathbb{U}, & \forall t \in [0, T] \\ x(0) = x_0, & x(T) = x_s \end{cases} \quad (1)$$

where x_0 is the initial condition and the pair (x_s, u_s) that satisfies

$$l_c(x_s, u_s) = \min_{x,u}\{l_c(x, u) \mid f(x, u) = 0, \quad x \in \mathbb{X}, \quad u \in \mathbb{U}\} \quad (2)$$

is defined as the optimal static equilibrium. The constraint sets $\mathbb{X} \subseteq \mathbb{R}^n$, $\mathbb{U} \subset \mathbb{R}^m$ with $\mathbb{X} \times \mathbb{U}$ assumed compact. This optimal control problem is at the core of model predictive control of systems where (1) is carried out in a receding horizon manner. The cost function $l_c(x(t), u(t))$ here is assumed generic. The direct design approach to solving (1) is to discretize it by the use of exact or approximate discretization methods, and then optimize.

As stated in Section 1, dissipativity of a system with respect to the given cost function is important in the context of e-MPC as the stability proof of the optimal controlled system often relies on this property. Hence, given a 'dissipative' continuous-time setup that is approximately discretized, it is imperative to know if dissipativity is preserved. A review of the literature on preservation of dissipativity shows that most of the work done is on finding supply rates for which the discretized system (controller) is passive while some viewed dissipativity preservation as preservation of positive-realness of the system [8–11]. However, dissipativity of the system as applied in e-MPC is with respect to a given running (stage) cost. Hence, given a continuous-time system that is dissipative with respect to a given supply rate, we seek to find out if this dissipativity is preserved in the discretized setup and in cases where the explicit discretized form is not available, how discretization affects the closed-loop system's stability.

For analysis purpose, we consider linear systems with quadratic running costs without restrictions on the definiteness of the cost. The origin is taken to be the optimal static equilibrium. The use of a linear-quadratic setup makes for ease of checking the dissipativity condition.

Definition 1. Consider the continuous-time system

$$\dot{x} = A_c x + B_c u \quad (3)$$

and the running cost

$$l_c(x, u) = x^T Q_c x + u^T R_c u + 2x^T S_c u \quad (4)$$

System (3) is said to be dissipative [12,13] with respect to running cost (4) if there exists a quadratic storage function, $V(x) = x^T P_c x$ where $P_c = P_c^T$ such that for all $x \in \mathbb{X}$, $u \in \mathbb{U}$,

$$\dot{x}^T P_c x + x^T P_c \dot{x} \le l_c(x, u). \quad (5)$$

This is implied by the existence of a $P_c = P_c^T$ such that the Linear Matrix Inequality (LMI)

$$\begin{bmatrix} A_c^T P_c + P_c A_c - Q_c & P_c B_c - S_c \\ B_c^T P_c - S_c^T & -R_c \end{bmatrix} \le 0 \quad (6)$$

is feasible. If (6) holds with strict inequality, the system is said to be strictly-dissipative with respect to the running cost.

Definition 2. The discrete time system

$$x_{k+1} = Ax_k + Bu_k \quad (7)$$

is said to be dissipative [14,13,5] with respect to the stage cost

$$l_d(x_k, u_k) = x_k^T Q x_k + u_k^T R u_k + 2x_k^T S u_k \quad (8)$$

if there exists a quadratic storage function $x_k^T P_d x_k$ where $P_d = P_d^T$ such that for all $x_k \in \mathbb{X}$, $u_k \in \mathbb{U}$ and $k \ge 0$,

$$x_{k+1}^T P_d x_{k+1} - x_k^T P_d x_k \le l_d(x_k, u_k) \quad (9)$$

This is implied by the existence of a $P_d = P_d^T$ such that the LMI

$$\begin{bmatrix} A^T P_d A - P_d - Q & A^T P_d B - S \\ B^T P_d A - S^T & B^T P_d B - R \end{bmatrix} \le 0 \quad (10)$$

is feasible. If (10) holds with strict inequality, the system is said to be strictly-dissipative.

We note that compactness of the constraint set and continuity of the quadratic storage function imply lower boundedness of the storage function, which is required for dissipativity to hold. Hence, P_c and P_d can be non-negative, provided the storage function remains lower bounded [4,5,15–17].

Assumption 2.1. The continuous-time system (3) is dissipative with respect to the running cost (4).

3. Effect of sampling period on first order approximation

In this section, we analyze the effect of the approximate discretization of optimal control problem (1) when the dynamics is linear and the cost function is quadratic. We note that there are established methods of computing the exact discrete equivalent of (1) when dealing with a linear-quadratic setup [18,19]. Hence while one may not necessarily use an approximate method in the linear-quadratic case, analyzing the effect of approximate discretization in the linear quadratic case will help understand the observed behaviour in the generic case. We start with the exact discretization method under the assumption of piecewise-constant inputs and take its first order approximation. This gives an explicit form of the discrete setup in terms of the continuous-time setup hence analysis linking both can be easily made.

Consider the continuous-time system (3) and running cost (4). Define

$$\Phi(t) = e^{A_c t}, \quad \Gamma(t) = \int_0^t e^{A_c \eta} B_c \, d\eta.$$

Given a sampling time $t_s > 0$, let (7) and (8) in Definition 2 be obtained by the exact discretization of (3) and (4) (under the assumption of piecewise-constant inputs i.e., zero-order-hold (ZOH)) such that:

$$A = \Phi(t_s) = I_{n_x} + A_c t_s + \frac{1}{2!} A_c^2 t_s^2 + \frac{1}{3!} A_c^3 t_s^3 + \cdots$$
$$B = \int_0^{t_s} e^{A_c t} B_c \, dt = B_c t_s + \frac{1}{2!} A_c B_c t_s^2 + \frac{1}{3!} A_c^2 B_c t_s^3 + \cdots \quad (11)$$
$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} = \int_0^{t_s} \begin{bmatrix} \Phi^T(t) & 0 \\ \Gamma^T(t) & I_{n_x} \end{bmatrix} \begin{bmatrix} Q_c & S_c \\ S_c^T & R_c \end{bmatrix} \begin{bmatrix} \Phi(t) & \Gamma(t) \\ 0 & I_{n_x} \end{bmatrix} dt$$

where n_x is the number of states. Expanding the discrete cost from (11) in powers of t_s yields the series expansion;

$$Q = Q_c t_s + \frac{1}{2}(Q_c A_c + A_c^T Q_c) t_s^2 + \cdots$$
$$R = R_c t_s + \frac{1}{2}(S_c^T B_c + B_c^T S_c) t_s^2 + \cdots \quad (12)$$
$$S = S_c t_s + \frac{1}{2}(A_c^T S_c + Q_c B_c) t_s^2 + \cdots$$

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