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Short communication

Mixed convection flow of thermally stratified MHD nanofluid over an exponentially stretching surface with viscous dissipation effect

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ABSTRACT

The present analysis concentrates to examine the influence of both thermal and solutal stratification on magneto-hydrodynamics (MHD) nanofluid flow along an exponentially stretching sheet. Moreover, simultaneous effects of mixed convection and viscous dissipation are also analyzed to determine the thermal conductivity within the restricted domain. Energy and concentration equation consist of two important slip mechanisms, namely: the Brownian motion of nanoparticles and the thermophoresis due to concentration difference. By the mean of compatible similarity transformed, a system of PDEs is converted into the system of nonlinear ODEs. The resulting nonlinear ODEs are successfully solved via the implicit finite difference method (FDM). Obtained numerical solutions are plotted for each profile for different and converging values of including parameters. To validate the results, numerical values of Nusselt number are compared with the existing literature for a particular case. Obtained results present the significant impact of each parameter on temperature and concentration. Nanofluid flow behaviour is also observed via velocity profile.

1. Introduction

Fluid flow past along a rough or flat surfaces attain a considerable attention by virtue of its extensive and considerable applications in engineering and manufacturing processes. Few important and foremost useful examples related to applications include the polymer extrusion, wires and fibre undercoat, food stuff wrapping, manufacturing of bags and papers, and petroleum manufacturing goods. Initially, Sakiadis [1] described the idea of boundary layer theory along a uniform moving surface that deals the free stream velocity and ambient fluid temperature is considered to be zero. Later on, Crane [2] has been examining the work for linear stretching velocity and it is proportional to the distance from the fixed slit. Thereafter the revolutionary idea presented by Crane has been extended by several other researchers by considering the various effects of heat and mass flow [3–5]. In the current decay, the most significant part of literature of fluid models is dealing with the linear stretching surface with various physical phenomena. However, Gupta and Gupta [3] established that all the physical phenomena need not to be dealt with linear but it can be dealt with the exponential or non-linear stretching for both heat and mass transfer. They have further discovered that sheet can be considered as a permeable to deal the phenomena of suction and injection of the fluid.

Apart from all the fundamental problem of linear stretching sheet, existing literature also witnesses that flow examination due to exponentially stretching sheet is also an important factor in most of the manufacturing process. In the beginning, the idea of flow past over an exponentially stretching sheet is presented by Magyari and Keller [6]. They have found the numerical solution of fluid flow over a sheet that is stretched with exponential velocity that deals the phenomena of heat and mass transfer characteristics. In another study, Elbashbeshy [7] extend the idea of Magyari and Keller [6] for exponential stretching sheet to deal the heat transfer with suction/injection effect. Currently many researchers have considered different and important ideas to deal the flow over an exponentially stretching surface for both Newtonian and non-Newtonian fluid models [8–15].

In the recent decay, heat transfer is one of the essential key features in the energy development at the industrial level and manufacturing process of any equipment. Despite of that fact heat addition, removing or transfer from one place to another place during the manufacturing process is totally based upon the thermal performance of working fluid. In several cases: water, engine oil,

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Nomenclature

u, v	velocity components in x and y directions (m/s)
B	magnetic induction (A/m) tesla
U	stretching velocity (m/s)
U_0	reference velocity (m/s)
k	thermal conductivity (W/m k)
V_0	strength of suction (m/s)
T	temperature of the fluid (K)
T_w	wall temperature (K)
T_∞	ambient fluid temperature (K)
C	nanoparticle concentration (mol/m^3)
C_w	nanoparticle concentration at the stretching surface (mol/m^3)
C_∞	nanoparticle ambient concentration (mol/m^3)
M	Hartmann number
Pr	Prandtl number
Le	Lewis number
Nt	thermophoresis parameter
Nb	Brownian motion parameter
Ec	Eckert number
S	suction/injection parameter
D_B	Brownian diffusion coefficient (m^2/s)
D_T	thermophoresis diffusion coefficient (m^2/s)
a, b, c, d	are dimensional constants

Greek symbols

η	similarity variable
α	thermal diffusivity (m^2/s)
υ	kinematic viscosity of nanofluid (m^2/s)
σ	electrical conductivity of nanofluid (S/m)
μ	dynamic viscosity of nanofluid (kg /m s)
ρ_f	density of the fluid (kg/m^3)
$(\rho c)_f$	capacity heat of the fluid (kg /m^3 K)
$(\rho c)_p$	capacity heat of the nanoparticles (kg /m^3 K)
ε_1	thermal stratification parameter
ε_2	solutal stratification parameter

lubricants and other common working fluid have poor thermal conductivity as compare to the required conductivity at the industrial level. Incorporation of tiny particles within the working fluid plays an important to boost the poor thermal conductivity of base fluid. Choi [16] introduced a new class to raise the low thermal conductivity of any base fluid by dispersing tiny particles in the working fluids. Recently nanofluids have gained interest of researchers due its enhanced thermal conductivity property [17]. Buongiorno [18] identified seven slip mechanisms in which Brownian motion and thermophoresis are important factors. Some review studies concerning the analysis of nanofluids can be observed in [19–22]. Recently, Kuznetsov and Nield [23] extended the concept of Buongiorno [18] for boundary layer phenomena with the vertical wall. Later on, Khan and Pop [24] modify the concept of Brownian motion and thermophoresis for horizontal stretching surface. Currently, an extensive literature can be found for both Newtonian and non-Newtonian fluid with various physical models in the presence of nanoparticles [25–35].

Thermal and concentration stratification is one of the important and natural phenomena that occur due to the density difference. Most of the case, where the density of a material varies due to temperature difference or homogenous mixture changes its stats by providing the temperature then molecules having substantial density may accumulate at the base of the surface and molecules with low density switch and rise up at upper part of the layer. An experimental and analytical study has been done by several researchers [36–38] for the flow due to a heated surface occupied due to stratified fluid. Thermal stratification produce when a uniform release of thermal boundary layer into the medium. Similarity solution attained by Kulkarni et al. [39], describe the natural convection flow near a heated plate for thermally stratified liquid. Plenty of the authors worked on the stratification with various effects [40–44].

The core part of this work is to analyse the influence of thermal and solutal stratification on nanofluid that moves over an exponential stretching sheet. For this, we have extended the idea of Khan and Pop [24] and modify it for double stratification phenomena in the form of PDEs. Transformed ODEs are solved via Keller box scheme. Effects of main emerging parameters are discussed graphically that based upon physical influence at velocity, temperature profile and concentration profiles. To highlight the results at the surface, results are also obtained for skin friction, Nusselt number and Sherwood number.

2. Mathematical model and formulation

It is considered that nanofluid flow is steady, incompressible, viscous, and two dimensional that past over a stretching sheet placed along $x-$axis while fluid is restricted in the domain $y > 0$. It is further assume that sheet is stretched with the exponential velocity $U = U_0 e^{x/l}$. We ae further considering that sheet is permeable. Buoyancy forces are also considered for thermal and concentration to deal the double stratified phenomena. Since the surface is heated with the temperature $T_w(x)$ and is embedded in a thermally stratified medium, and assume that the variable temperature $T_w = T_0 + a e^{x/2l}$ and concentration $C_w = C_0 + b e^{x/2l}$ at the surface and away from the sheet. Fig. 1 depicts the physical explanation of the given model. A variable magnetic field $B(x) = B_0 e^{x/2l}$ is applied that is normal to the surface, B_0 being constant. The governing equations for the above flow are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

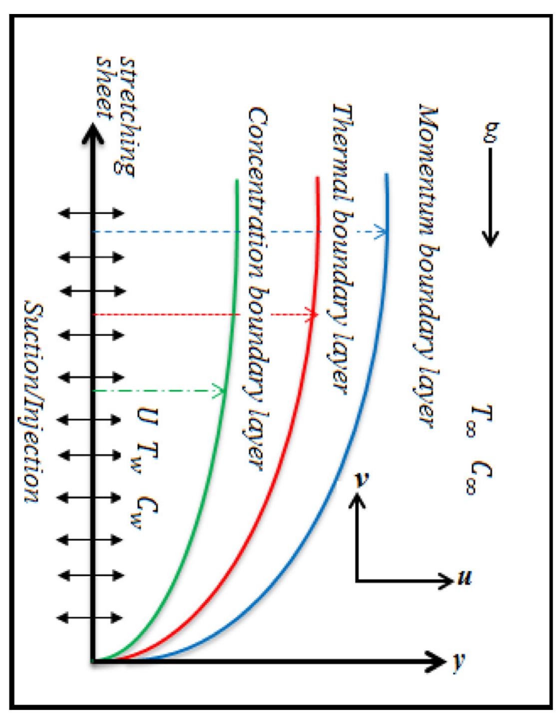

Fig. 1. Geometry of the model.

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