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Flow and heat transfer analysis of Jeffery nano fluid impinging obliquely over a stretched plate

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This study investigates the problem of oblique stagnation point flow using Jeffery nanofluid as a rheological fluid model. Effects of thermophoresis and Brownian motion are taken into account. The governing nonlinear partial differential equations for the flow field are obtained and then converted to ordinary differential equations via suitable transformations. Consequential highly non-linear system of differential equations is solved numerically through mid-point integration as a basic scheme along with Richardson's extrapolation as an enhancement scheme and analytical results are also obtained using optimal homotopy analysis Method (OHAM). Non-dimensional velocities, temperature and concentration profiles are expressed through graphs. Numerical values of local skin friction coefficients, local heat and mass flux are tabulated numerically as well as analytically for various physical parameters emerging in our flow problem. The obtained results revealed that both normal and tangential skin friction coefficients decrease with an increase in Jeffery fluid parameter. It is also observed that an enhancement in Thermophoresis and Brownian motion parameters leads to a reduction in heat flux at the wall. Comparison of numerical data is made with previous existing literature to confirm accuracy of present study for the case of Newtonian fluid.

1. Introduction

Understanding the nature and flow behaviour of non-Newtonian fluids are quite interesting area of research which is still not very convenient for scientists and engineers to explore it completely. Fluid flows with rheological characteristics are extremely important in many industrial processes. The problem of two-dimensional stagnation-point flow is among the classical problems in fluid dynamics. Numerous studies are dedicated to exploit the subject of stagnation point flows viewing its application in problems like coating of layers onto rigid materials, extrusion of plastic sheets and process like fabrication of adhesives [1,2]. Chiam [3] investigated stagnation point flow towards a stretching plate. In another article heat transfer analysis of stagnation point flow towards a stretching sheet was highlighted by Gupta et al [4]. Despite above mentioned studies, subject of oblique stagnation point flow has not been given much interest so far and not a great amount of literature is available on this particular topic. Although there are a few critical studies available *e.g.*, Kimiaeifar and Bagheri [5] investigated steady non-orthogonal stagnation point flow of a viscous and incompressible fluid. Pop et al [6] highlighted non-orthogonal stagnation-point flow towards a stretching surface in a non-Newtonian fluid with heat transfer. Interesting observations can be found from independent studies by Stuart [7], Tamada [8] and Dorrepaal [9] when fluid impinges obliquely on the plate. Mehmood et al [10] inspected non-orthogonal stagnation point flow of micropolar second grade fluid towards a stretching surface with heat transfer. Nanofluids are subject of global interest now a day. These solid nanoparticles with length scales of 1–100 nm are suspended in conventional heat transfer base fluid. They are assumed to be highly capable for enhanced thermal conductivity. Choi [11] introduced the idea of nanofluids in his investigation. Later on Putra et al [12] discussed natural convection of nanofluids through experimental investigations. Stagnation point flow of nanofluid over a stretching/shrinking sheet has been highlighted by Bachok et al [13]. Dual solutions of stagnation-point flow of nanofluid over a stretching surface are presented by Kameswara et al [14]. Nadeem et al [15] investigated axisymmetric stagnation

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Nomenclature

C	specific heat of fluid (kg m^{-2} K s^{-2})
k	thermal conductivity (W m^{-1} K^{-1})
ρ	density (kg m^{-3})
α	thermal diffusivity (m^2 s^{-1})
μ	dynamic viscosity (kg m^{-1} s^{-1})
T^*	temperature (K)
U, V	velocity comp- in x-y direction (m s^{-1})
f	dimensionless normal velocity
h	dimensionless tangential velocity
p	pressure
C^*	concentration

Subscripts

pf	pertaining to nanofluid
bf	pertaining to base fluid
w	pertaining to wall
∞	pertaining far from wall
Pr	Prandtl number
Sc	Schmidt number
K	Jeffery fluid parameter
Nb	Brownian motion parameter
Nt	thermophoresis parameter
θ	dimensionless temperature
ϕ	dimensionless concentration
D_T	thermophoresis coefficient
D_B	Brownian diffusion coefficient
λ_1	ratio of relaxation to retardation time
λ_2	retardation time

flow of a micropolar nanofluid in a moving cylinder. Very recently, Bhattacharyya [16] investigated heat transfer in boundary layer stagnation-point flow towards a shrinking sheet with non-uniform heat flux. In another article, Akgul et al [17] numerically discussed cooling intensification using several types of nanofluids over a continuously moving stretching surface. Das [18] presented the convective heat transfer performance of nanofluids over a permeable stretching surface in the presence of partial slip. Soleimani et al [19] examined MHD natural convection in a nanofluid filled inclined enclosure with sinusoidal wall using CVFEM. Sandeep and Sulochana [20] presented dual solutions of radiative MHD nanofluid flow over an exponentially stretching sheet with heat generation/absorption. Ellahi et al [21] explored Non Newtonian nanofluids flow through a porous medium between two coaxial cylinders with heat transfer and variable viscosity. Ganji et al [22] studied the effects of thermal radiation on nanofluid flow and heat transfer using two phase model. Recently Ellahi [23] analytically discussed MHD and temperature dependent viscosity effects on flow of non-Newtonian nanofluid in a pipe.

Saleem et. al. [24] presented buoyancy and metallic particle effects on an unsteady water-based fluid flow along a vertically rotating cone. Sheikholeslami et. al [25] explored ferrofluid flow and heat transfer in a semi annulus enclosure in the presence of magnetic source considering thermal radiation. Entropy analysis of radioactive rotating nanofluid with thermal slip was discussed by Rehman et. al [26]. Ellahi et. al [27] developed series solutions of non-Newtonian nanofluids with Reynolds' model and Vogel's model by means of the homotopy analysis method. Nadeem and Saleem [28] examined analytical treatment of unsteady mixed convection MHD flow on a rotating cone in a rotating frame. Mixed convective oblique flow of a Casson fluid with partial slip, internal heating and homogeneous-heterogeneous reactions were presented by Rana et. al [29]. Sheikholeslami et. al [30] explained simulation of MHD CuO–water nanofluid flow and convective heat

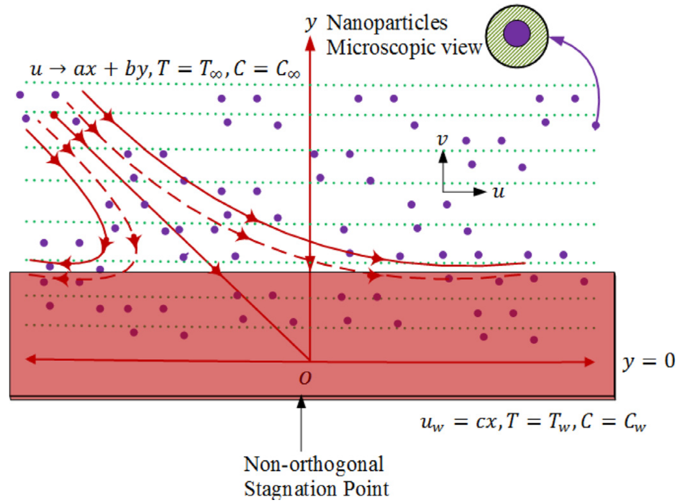

Fig. 1. A physical description of the problem.

transfer considering Lorentz forces. Rashidi et. al [31] described lie group solution for free convective flow of a nanofluid past a chemically reacting horizontal plate in a porous media. Shape effects of nanoparticles suspended in HFE-7100 over wedge with entropy generation and mixed convection were investigated by Ellahi et. al. [32]. Some more useful studies related to current topic can be found in [24–47]. In this study we tried to examine flow behaviour of rheological Jeffery fluid over a stretching surface in the presence of nanoparticles when fluid strikes the surface obliquely. The governing system of partial differential equations is transformed to couple ordinary differential equations using similarity transformations. These governing equations are solved analytically using Optimal HAM [34–37]. In order to validate our analytical results through OHAM, numerical results are also obtained using mid-point integration scheme along with Richardson extrapolation using computational software Maple [39–43]. Physical quantities of interest such as local skin frictions, local heat and mass flux are tabulated. The key findings of the article are concluded at the end of the manuscript.

2. Mathematical formulation

Consider steady two-dimensional oblique stagnation point flow of a Jeffery fluid over a stretching surface. Two equal and opposite forces are applied along $x-axis$ so that surface is kept stretched and origin fixed as shown in Fig. 1.

The effects of Brownian motion and thermophoresis are also incorporated. The resulting equations of flow, temperature and concentration in the presence of nanoparticles are presented as [34]

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0, \tag{1}$$

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} + \frac{1}{\rho f}\frac{\partial p^*}{\partial x^*} = \left(\frac{\nu}{1+\lambda_1}\right)\left[\nabla^{*2}u^* + 2\lambda_2\frac{\partial}{\partial x^*}\left(u^*\frac{\partial^2 u^*}{\partial x^{*2}} + v^*\frac{\partial^2 u^*}{\partial x^*\partial y^*}\right) + \lambda_2\frac{\partial}{\partial y^*}\left\{u^*\left(\frac{\partial^2 u^*}{\partial x^*\partial y^*} + \frac{\partial^2 v^*}{\partial x^{*2}}\right) + v^*\left(\frac{\partial^2 v^*}{\partial x^*\partial y^*} + \frac{\partial^2 u^*}{\partial y^{*2}}\right\}\right], \tag{2}$$