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Fuzzy robust optimization for handling feed stream and model parameter uncertainties during comminution process

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ABSTRACT

Uncertainty analysis of an industrial grinding optimization process involving various sources of uncertainties and multiple numbers of objective functions has been studied in this work. Two primary sources of uncertainties, *e.g.* (1) operational parameters such as feed size distributions that are subjected to uncertainty due to varied range of feed sources that the industrial grinding process handles, and (2) model parameters that are obtained by the regression analysis of experimental data and subjected to regression and experimental errors, have been considered. Moreover, obtaining statistical distributions for these uncertain parameters are practically challenging. Hence, these parameters are considered as fuzzy numbers and the embedded uncertain multi-objective optimization problem has been analyzed using credibility based fuzzy robust optimization (FRO) technique. This technique helps in converting the uncertain fuzzy optimization formulation into a deterministic equivalent form, which can be further utilized to obtain the Pareto solutions by well-developed evolutionary algorithms. Initially, PO solutions are obtained by considering the uncertainty in different sets of uncertain parameters separately. This is followed by the study of amalgamated effect of uncertain parameters on PO solutions. Along with this, different fuzzy measures *e.g.* credibility, possibility, necessity *etc.* are utilized to observe their effects on final solutions. PO solutions obtained from possibility and necessity based FRO show the optimistic and pessimistic attitudes of risk, respectively. This provides a key to a decision maker to select any point based on the existing risk appetite. As compared to the deterministic results, the robust Pareto solutions show potential improvements of 26% and 16% in throughput and mid-size fractions, respectively. This generic approach not only provides the solution for robust range of operation based on the risk appetite of the enterprise, but also helps a decision maker to decide which parameters, amidst the set of uncertain parameters, are more sensitive to the results utilizing various fuzzy measures such as credibility, possibility and necessity. Along with this, the PO solutions obtained with robust optimization are compared with the solutions obtained by Expected Value Model (EVM).

1. Introduction

One of the reasons behind the popularity of building a mathematical model of a chemical process is its ability to simulate various scenarios and thereby saving huge cost and time behind conducting each one of them experimentally. Mathematical models thus built often have a set of parameters embedded in them that are generally determined by tuning the model with experimental results to mimic the model behavior close to the process conditions. As these model parameters are regressed from the experimental data, they are subjected to uncertainty related to the experimental and regression errors; omission of such errors could lead to unrealistic results [1]. For example, while conducting optimization based studies using one of these models, the formulations are most frequently established in a deterministic fashion where it is ensured that all parameters appearing in the optimization formulation, other than the decision variables, are certainly known and their values cannot be changed during the entire course of optimization [2]. These parameters include the coefficients in the objective function as well as constraints *e.g.* coefficients A, b and c in an optimization formulation of type **Min** $\mathbf{c}\mathbf{x}$, subjected to $A\mathbf{x} \leq b$. However, if some of these parameters are exposed to real life uncertainty, the results obtained assuming them as constants may lead to infeasibility when they are changed even slightly from their existing values [3]. Conventionally, these uncertainties can be handled by overdesigning the equipment or overestimating the operational parameters and thereby absorbing the uncertainty embedded in them. Another popular approach is to replace the uncertain parameters with their nominal values and

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solve the deterministic formulation. These ideas are not pragmatic because the former leads to a very conservative solution where full potential of the plant/equipment is never utilized, whereas in case of the latter, the benefit of the changing operating conditions can never be exploited leading to either missing opportunities or unnecessary inventory piling due to under and over production situations. The paradigm of optimization under uncertainty (OUU) has, therefore, become popular as it allows a decision maker to create reliable solutions that remain feasible in the presence of parametric uncertainty.

Broadly, two types of OUU methodologies exist in the literature [4]. The first one, the probability based approaches, assumes that the uncertain parameters have to follow some statistical distribution. Stochastic programming (SP), chance constrained programming (CCP) and, expected value model (EVM) [4] belong to this category. Amidst them, the most popular is the scenario based two-stage SP (TSSP) [5] technique, where the decision variables can be deployed in two stages, one before the realization of uncertainty ("here and now") and the other after the realization ("wait and see"). As a result, the objective function, represented as the sum of the first stage components and expectation of second stage cost components, involving both first and second stage decision variables. One of the major drawbacks of this process is that the problem size increases exponentially as the numbers of uncertain parameters and scenarios increase, thus leading to an uncontrollable situation under the finite computational resources. Apart from this, it may not be easy always to decompose the problem into two sets of decision variables. Unlike SP, CCP, first introduced by Charnes and Cooper [6], provides a competitive way to solve OUU problems in terms of problem size and execution time management. This technique introduces the probability of constraint satisfaction, which can be connected to the reliability of the solution obtained, assuming that a constraint may not be satisfied for all uncertainty realizations. To make the problem more tractable, the probabilistic formulation is converted into an equivalent deterministic optimization problem (EDOP) formulation, which can be solved by conventional optimization techniques. The size of the EDOP is generally manageable even in the presence of a large number of uncertain parameters unless the probability of constraint satisfaction is set to be very high (*i.e.* very close to unity). Conversely, the EVM uses expected values of the objective function and the constraints. In a nutshell, the assumption that the uncertain parameters have to follow some well-behaved, known probability distributions and that can be estimated quite accurately in the above approaches usually requires an astronomical, completely unrealistic number of observations and might not be a practical task under real life situations [3].

On the contrary, fuzzy based approaches [7] help to avoid most of the afore-mentioned lacuna. Expressing the uncertain parameters as fuzzy variables, the extent of constraint violation can be represented by defining a membership function between 0 and 1, where 0 signifies maximum extent of constraint violation and 1 signifies no violation. Intermediate violations can be interpolated using different kinds of linear or non-linear fuzzification approaches without much restrictions. Fuzzy chance constrained programming (FCCP) [8] uses possibility of constraint satisfaction instead of probability measure in CCP. Similarly, in fuzzy expected value model (FEVM), expected values are calculated using possibility measure. Along with possibility, other fuzzy measures, *e.g.* necessity, credibility *etc.*, representing various attitudes of a decision maker, can also be used to analyze the uncertain scenarios. Dubois and Prade [9] defined fuzzy variable as a mapping element from the space of possibility or necessity to the space of real numbers. To define an intermediate state of decision making and overcome the drawback of not owning the property of self-duality, Liu [10] introduced the credibility measure, defined as a weighted average of the possibility and necessity measures. Applications of credibility based FCCP and FEVM are available in the literature [11–13]. However, going by the very definition of a robust solution, which is a fixed decision variable vector that should remain feasible irrespective of the realization of the uncertainty in the parameters [3], the solutions obtained by above approaches may not be robust. So, robust optimization is an alternative way to handle OUU problems and might be extremely important under situations where the cases of constraint violation are strictly restricted. Credibility based fuzzy robust optimization is one technique which can be used to transform the uncertain optimization problem into its equivalent deterministic optimization problem.

In mineral processing industries, grinding is one of the most popularly used size reduction processes. However, it is highly energy-intensive process, where any possibility for reduction of energy consumption can provide both economic edge and carbon footprint advantages. Thus, modeling and optimization of grinding have been of utmost importance for finding out the best operating conditions. Significant work have been done by several researchers [14–15] on modeling of grinding circuit. As per these works, the grinding circuit comprises operations such as rod mill, ball mill *etc.*, as well as separation units such as hydro-cyclones, where modeling of the grinding operation is the most crucial. Bond [16] attempted to summarize the calculation methods in the grinding process using power based models. Despite being advantageous in terms of prediction accuracy as well as simplicity in approach, these models suffer from extrapolation across various dimensions of design changes. Austin et al. [17] and Morrell et al. [18] presented advanced modeling concept by introducing population balance methods encompassing selection functions, breakage functions and discharge functions. These parameters allow the designer to monitor and track particle breakage and transition within several size classes with time inside the grinding mill. While exploiting the improvements in the computational power, Chakraborti et al. [19,20] investigated the modeling of particle–particle interactions and particle flow patterns inside the grinding unit using computational fluid dynamics. Mishra [21], Mishra and Rajamani [22] and Datta and Rajamani [23] further analyzed particle–particle interaction behavior by discrete element modeling method. For the above computational techniques, the decision maker has to specify the grid points, boundary conditions and a set of starting points. Based on the starting solutions, states of the particle are calculated at all grid points honoring the boundary conditions and accurate results are obtained at the cost of computational time and rigor. Similarly, modeling of hydro-cyclones was presented by several researchers [24,25], which have been successful in predicting the split of the material coming inside the hydro-cyclone unit as a feed into the overflow and underflow streams. Many of these unit operations are considered simultaneously in an integrated grinding circuit, where a population balance approach can be used for simulating the circuit performance [26–28] and time dependent behavior of different size classes can be predicted within reasonable computational time. Wei and Craig [15] conducted a survey on the development of control philosophy of grinding where various sets of manipulated variables (mill solids flow rate, sump and mill water flow rate, sump slurry flow rates), control variables (cyclone overflow product density, feed ratio, slurry level in the sump, cyclone feed pressure, product particle size) and control objectives (throughput maximization) were surfaced. PID controllers are predominately used in the grinding mills. Though the studies on optimization are ubiquitous among various fields [29–32], most of them are deterministic in nature, where the effect of uncertain parameters on the optimization results is ignored for the sake of simplicity.

In this work, credibility based fuzzy robust optimization (FRO) technique has been applied to an industrial grinding case study