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Unsteady MHD flow and radiation heat transfer of nanofluid in a finite thin film with heat generation and thermophoresis

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ABSTRACT

This paper presents an investigation for unsteady MHD flow and radiation heat transfer of a nanofluid in a finite thin film over stretching surface in which the effects of heat generation, thermophoresis and Brownian motion are taken into account. Boundary layer governing differential equations are formulated and reduced into a set of ordinary differential equations by suitable similarity transformations. Solutions are obtained numerically and some interesting results are found. Results show that the film thickness decreases monotonically with unsteady parameter and the magnetic parameter increase but increases with the power law index number *m*. The temperature profile decreases while the nanoparticle volume fraction increases as the thermophoresis parameter increases. More effects of involved parameters on velocity, temperature and concentration fields are graphically presented and analyzed in detail.

1. Introduction

Recently, the analysis of the flow and heat transfer [1,2] on a thin liquid film over stretching surface has attracted considerable attention due to its wide applications. Such applications include wire and fiber coating, metal and polymer extrusion, foodstuff processing, continuous casting, drawing of plastic sheets, exchangers, transpiration cooling, reactor fluidization, chemical processing equipment, etc. All coating processes demand a smooth glossy surface to meet requirements for appearance, low friction, transparency and strength. The knowledge of flow and heat transfer within a thin liquid film is of significant importance in understanding the coating process and design of chemical processing equipment and various heat exchangers.

The earlier research work on considering the steady two-dimensional flow of a Newtonian fluid driven by a stretching elastic flat sheet was carried out by Crane [3]. Following his pioneering work, many authors explored various aspects of the flow and heat transfer occurring in a thin liquid film surrounding the stretching sheet. The flow problem within a finite film of Newtonian fluid over an unsteady stretching sheet was carried out by Wang [4] who analyzed asymptotic and numerical solutions. The momentum and heat transfer in a laminar liquid film on a horizontal stretching sheet had been explored by Andersson et al. [5,6] and Liu et al. [7]. Further, Chen [8] had considered viscous dissipation and Marangoni effects on heat transfer of power law fluid in thin liquid film on an unsteady stretching surface. Wang [9] and Wang and Pop [10] used homotopy analysis method to study a laminar liquid film on a horizontal stretching sheet. They also derived the critical value involved unsteadiness parameter. Vajravelu et al. [11] studied the problem of flow and heat transfer in a liquid film of a non-Newtonian Ostwald-de Waele fluid over a horizontal porous stretching surface in the presence of viscous dissipation and temperature-dependent thermal conductivity. Dandapat and Ray [12] studied the thin liquid film flow over a rotating horizontal disk. Kumari and Nath [13] considered the unsteady MHD film over a rotating infinite disk and taken into account the effect of the axial magnetic field. Later, Dandapat et al. [14] analyzed a liquid film on an unsteady stretching surface in the absence of thermocapillary effects. Noor and Hashim [15] studied the effects of thermocapillarity and a magnetic field by HAM. Abbas et al. [16] proposed HAM to discuss the flow problem in a thin liquid film of second grade fluid over an unsteady stretching surface. Abel et al. [17] presented a mathematical analysis to a laminar liquid film from a horizontal stretching surface. Rashidi and Keimanesh [18] studied the MHD flow in a laminar liquid film from a horizontal stretching surface by Khalili et al. [19] studied the unsteady convective heat and mass transfer in pseudoplastic nanofluid over a stretching

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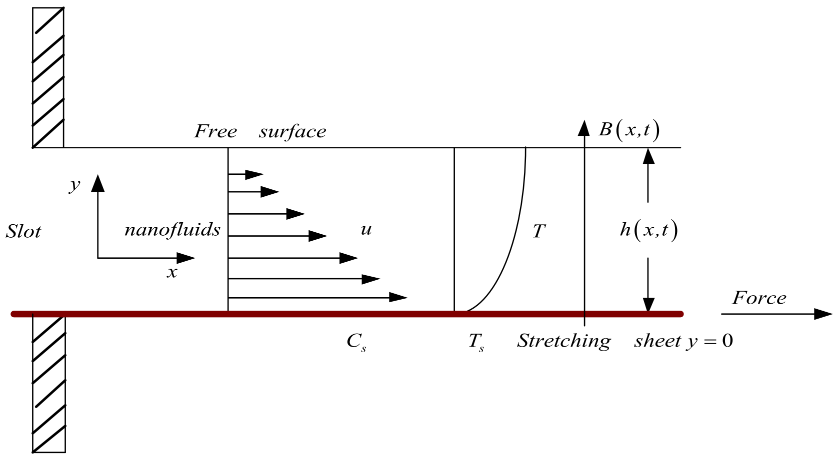

Fig. 1. Schematic of the physical model.

wall. Bachok et al. [20] and Xu et al. [21] extended the problem of flow and heat transfer within a finite thin film over an unsteady stretching sheet. Aziz et al. [22,23] investigated effects of viscous dissipation and heat source on the heat transfer of thin film with finite thickness, and considered the surface temperature of more extensive significance. Lin et al. [24] studied flow and heat transfer of MHD pseudo-plastic nanofluid in a finite film over unsteady stretching surface taking into account the internal heating effects. A list of the literature concerning the flow and heat transfer on a thin liquid film over stretching surface can also be found in Refs. [25–28].

There are currently some interesting studies on the nanofluid flow. Sheikholeslami et al. [27] studied the effect of thermal radiation on magnetohydrodynamics nanofluid flow between two horizontal rotating plates. The influence of non-uniform electric filed on Fe3O4-Ethylene glycol nanofluid hydrothermal treatment in an enclosure with sinusoidal upper and moving lower walls is investigated by Sheikholeslami and Ellahi [28]. Sheikholeslami et al. [29] also analyzed the unsteady flow of a nanofluid squeezing between two parallel plates. Hayat et al. [30] analyzed the mixed convective peristaltic transport of water based nanofluids using five different nanoparticles.

Motivated by above mentioned works, we consider the flow of a nanofluid in a finite thin film over an unsteady stretching sheet. The effects of thermal radiation [31] and heat generation on the heat and mass transfer coefficients [32] are taken into account. We also consider magnetic field [33,34] on the flow and heat transfer of fluids within a finite thin liquid film on an unsteady stretching sheet in the governing equations. The set of ordinary differential equations are obtained by the semi-similarity transformation [32,35] and then are solved numerically by using bvp4c from Matlab. The influence of Brownian number, thermophoresis number, Prandtl number, Lewis number, magnetic parameter and internal heating parameter are presented graphically and discussed.

2. Formulation of problem

2.1. Governing equations and boundary conditions

Consider the unsteady flow and heat transfer of nanofluid [36,40] in finite thin liquid films past a stretching sheet. The thin elastic sheet which emerges from a narrow slit at the origin of a Cartesian co-ordinate system is shown in Fig. 1.

The problems are strongly nonlinear which are complex to study analytically or numerically. For simplicity, in present paper, the special emphasis is given to the formulation of the equations which provide similarity solutions. In doing so, we make the assumptions as following:

The continuous sheet moves with the velocity

$$U(x,t) = \frac{U_0 x^m}{1-ct} \tag{1}$$

where U_0 and c are positive constants. The surface temperature T_s of the stretching sheet with the distance x is defined as

$$T_s = T_0 - T_{ref}\frac{bx^{2m}}{1-ct}, \tag{2}$$

where T_0 is the temperature at the slit and T_{ref} is assumed as a constant reference temperature. And the temperature distribution is $T_w(x,t)$. A thin elastic liquid film of uniform thickness $h(x,t)$ lies on the horizontal stretching sheet. We assume that the stretching of the elastic sheet has internal heat generation/absorption and the surface of the liquid film is smooth. A magnetic field $B = B_0 x^{(m-1)/2}/\sqrt{1-ct}$ is imposed on the fluid flow in the y-direction. Moreover, the concentration C_s of the stretching sheet is assumed to vary with the distance x from the slit as

$$C_s = C_0 - C_{ref}\frac{bx^{2m}}{1-ct} \tag{3}$$

where C_0 is the temperature at the slit and C_{ref} is the reference concentration. It is assumed that the nanofluid is incompressible and the flow is laminar.

The continuity, momentum, energy and concentration equations for the fluid using the above assumptions with the Oberbeck-Boussinesq and the boundary layer approximations can be written as [37,38]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{4}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho_f}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2 u}{\rho_f}, \tag{5}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + Q(t) + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_0}\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{6}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_0}\frac{\partial^2 T}{\partial y^2}. \tag{7}$$

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