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Study of nanofluid flow and heat transfer between non-parallel stretching walls considering Brownian motion

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ABSTRACT

In this paper, the nanofluid flow and heat transfer between non-parallel stretching walls with Brownian motion effect is investigated. The governing radial momentum and energy equations are solved by Duan–Rach Approach (*DRA*). This method allows us to find a solution without using numerical methods to evaluate the undetermined coefficients. This method modifies the standard Adomian Decomposition Method by evaluating the inverse operators at the boundary conditions directly. The effective viscosity and thermal conductivity of nanofluid are calculated via *KKL* (Koo–Kleinstreuer–Li) correlation in which influence of Brownian motion on the thermal conductivity is considered. The effects of various parameters such as the stretching/shrinking parameter, the radiation parameter, Reynolds number, the opening angle and the heat source parameter are investigated on the velocity and temperature. Also, the value of the Nusselt number is calculated and presented through figures. The results show that the fluid velocity, temperature profile and Nusselt number increase with the increasing of stretching parameter. The results also reveal that the temperature profile increases with the increasing of the heat source parameter and it decreases with the rising of radiation parameter for both divergent and convergent channel. In addition, the results were compared with the previous works and found proposed method has high accuracy to solve this nonlinear problem.

1. Introduction

The fluid flow and heat transfer between parallel/non-parallel plates have received notable consideration in recent years as they are most applicable in the mechanical engineering and industrial applications. One of the first studies of this type of flow can be traced back to Jeffery [1] and Hamel [2]. There are newly developed methods such as Collocation Method [3], Differential Transformation Method [4–10], Homotopy Analysis Method [11–14], Adomian Decomposition Method [15–17], Homotopy Perturbation Method [18–20] and Optimal Homotopy Analysis Method [21] to solve the equation of Jeffery–Hamel fluid flow and many nonlinear problems in the engineering field. The unsteady motion of a rigid spherical particle in a quiescent shear-thinning power-law fluid was studied by Rahimi-Gorji et al. [3]. Their results show that the time of reaching the terminal velocity in a falling procedure significantly increases with growing of the particle size. The falling of a spherical particle in a plane Couette Newtonian fluid flow was studied by Dogonchi et al. [6]. They applied the DTM-Padé to solve governing equations. They concluded that the horizontal and vertical velocities of spherical particles in water fluid are higher than the ethylene-glycol and glycerin fluids. Dogonchi et al. [7] studied the motion of a vertically falling non-spherical solid particle in an incompressible Newtonian fluid. They applied the DTM-Padé to solve governing equations. They observed that the velocity of the gold particle is higher than the aluminum and copper particles. The entropy generation for the *MHD* nanofluid flow past a stretching permeable surface was analyzed by Abolbashari et al. [11]. Their results show that the rising of the unsteadiness parameter, nanoparticle volume fraction parameter, magnetic parameter, Reynolds number, suction parameter, Hartmann number and Brinkman number lead to an increase of the entropy generation number. Rashidi and co-workers [12] studied the heat transfer for *MHD* viscoelastic fluid flow over a porous wedge with thermal radiation effect. They found that the heat transfer to the fluid increases with the increasing value of the wedge angle. The effect of thermal radiation on the classical Jeffery–Hamel flow from a source or sink in convergent/divergent channels was studied by Barzegar Gerdroodbary et al. [22]. Their results show that the temperature profile increases with the rising of thermal radiation parameter. The classical Jeffery–Hamel flow from a source or sink in

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Nomenclature

A_1, A_2, A_3, A_4	constant parameter
C	stretching/shrinking parameter
Ec	Eckert number
C_p	specific heat (J/kg K)
f	dimensionless velocity
Θ	dimensionless temperature
P	pressure term
Re	Reynolds number
Pr	Prandtl number
s	stretching/shrinking rate (m^2/s)
$q_{rad.}$	radiative heat flux
T	temperature (K)
T_w	surface temperature of channels (K)
T_∞	free stream temperature
r,θ	cylindrical coordinates
u	velocity component in the radial direction (m/s)
u_c	rate of movement in the radial direction (m^2/s)
u_w	wall velocity component in the radial direction (m/s)
Hs	heat source parameter
N	radiation parameter
Nu	Nusselt number
C_f	skin fraction coefficient
DRA	Duan–Rach approach
ADM	Adomian decomposition method
k	thermal conductivity
$k*$	mean absorption coefficient
R_f	thermal interfacial resistance
d_p	particle size

Greek symbols

α	angle of the channel
η	dimensionless angle
σ	electrical conductivity
θ	any angle
ρ	density
μ	dynamic viscosity
ν	kinematic viscosity
ϕ	solid volume fraction
σ^*	Stefan–Boltzmann constant

Subscripts

nf	nanofluid
f	base fluid
p	nano-solid-particles

convergent/divergent channels with stretchable walls was investigated by Turkyilmazoglu [23]. He observed that the temperature profile increases with the rising of stretching parameter for the convergent channel.

The primary obstacle to enhance the heat transfer in engineering systems is the low thermal conductivity of customary fluids such as water, air, oil, and ethylene glycol mixture. Solid usually has a higher thermal conductivity than liquids. Addition of Nanoparticles to the conventional fluid, the so called "Nano-fluid", can improve the thermal conductivity of the mixture. Recently, several authors studied nanofluid flow and heat transfer [24–45]. Rashidi et al. [24] studied heat transfer of nanofluid flow over a stretching sheet in the presence of transverse magnetic field considering thermal radiation and buoyancy effects. They concluded that the increasing of buoyancy parameter increases the velocity profile and decreases the temperature profile of the nanofluid. The effect of magnetic field on the nanofluid flow inside a sinusoidal two-tube heat exchanger was investigated numerically by Valiallah Mousavi et al. [25]. They applied finite volume method to solve governing equations. Their results show that the sinusoidal formation of the internal tube significantly increases the Nusselt number inside a two-tube heat exchanger. Ferrofluid flow and heat transfer in the presence of an external variable magnetic field were studied by Kandelousi [26]. His results indicate that the Nusselt number is an increasing function of Magnetic number, Rayleigh number and nanoparticle volume fraction while it is a decreasing function of the Hartmann number. The influence of magnetic field dependent (MFD) viscosity on free convection heat transfer of nanofluid in an enclosure was studied by Sheikholeslami et al. [27]. They applied Control Volume based Finite Element Method to solve governing equations. They observed that the Nusselt number is an increasing function of Rayleigh number and volume fraction of nanoparticle while it is a decreasing function of viscosity parameter and Hartmann number. The effect of an electric field on Fe_3O_4–water nanofluid flow and heat transfer in a channel was studied by Safarnia et al. [28]. They found that the Nusselt number has a direct relationship with the Reynolds number and voltage supply. Nanofluid flow and heat transfer over a stretching porous cylinder considering thermal radiation was investigated by Sheikholeslami et al. [29]. They proved that skin friction coefficient increases with increase of Reynolds number and suction parameter, but it decreases with increase of the nanoparticle volume fraction. Effect of electric field on the hydrothermal behavior of nanofluid in a complex geometry was studied by Sheikholeslami et al. [30]. Their results show that supplied voltage can change the flow shape. Impact of non-uniform magnetic field on nanofluid hydrothermal treatment considering Brownian motion and thermophoresis effects was investigated by Sheikholeslami and Rashidi [31]. They reported that the Nusselt number has a direct relationship with Rayleigh number, buoyancy ratio number and Lewis number while it has a reverse relationship with Hartmann number. The effect of non-uniform magnetic field on nanofluid forced convection heat transfer in a lid driven semi-annulus considering Brownian motion was studied by Sheikholeslami et al. [32]. Force convection heat transfer in a lid driven semi annulus enclosure in presence of non-uniform magnetic field was studied by Sheikholeslami et al. [33]. They applied control volume based finite element method (CVFEM) to solve the governing equations. The effect of spatially variable magnetic field on ferrofluid flow and heat transfer was investigated by Sheikholeslami and Rashidi [34]. They applied control volume based finite element method (CVFEM) to solve the governing equations. Their results indicate that the Nusselt number is an increasing function of Magnetic number, Rayleigh number and nanoparticle volume fraction, while it is a decreasing function of Hartmann number. Effect of Lorentz forces on forced-convection nanofluid flow over a stretched surface was studied by Sheikholeslami et al. [35]. Their results illustrate that the coefficient of skin friction enhances with enhancing magnetic parameter while reduces with enhancing velocity ratio parameter. Ferrofluid flow in a semi annulus enclosure considering radiative heat transfer was studied by Sheikholeslami et al. [36]. Their results show that the Nusselt number is a rising function of Rayleigh number and Magnetic number while it is a decreasing function of radiation parameter. *MHD* convective nanofluid flow over a vertical stretching sheet considering the thermal radiation and buoyancy effects was studied by Pourmehran et al. [42]. Their results indicate that the reduced Nusselt number has an inverse relation with increasing the nanoparticle concentrations. An analytical investigation of the heat transfer for the microchannel heat sink (MCHS) cooled by different nanofluids (Cu, Al_2O_3, Ag, TiO_2 in water and ethylene glycol as base fluids) was studied by Rahimi-Gorji et al. [43]. They reported that by increasing the nanoparticles volume fraction, the Brownian