ARTICLE IN PRESS
JID: JARAP [m5G;March 11, 2017;20:9]
Annual Reviews in Control 000 (2017) 1–16
Contents lists available at ScienceDirect
Annual Reviews in Control
journal homepage: www.elsevier.com/locate/arcontrol

Review

A tutorial on modeling and analysis of dynamic social networks. Part I[☆]

Anton V. Proskurnikov[a,b,c,*], Roberto Tempo[d]

[a] Delft Center for Systems and Control, Delft University of Technology, Delft, The Netherlands
[b] Institute of Problems of Mechanical Engineering of the Russian Academy of Sciences (IPME RAS), St. Petersburg, Russia
[c] ITMO University, St. Petersburg, Russia
[d] CNR-IEIIT, Politecnico di Torino, Torino, Italy

ARTICLE INFO

Article history:
Received 17 January 2017
Revised 5 March 2017
Accepted 5 March 2017
Available online xxx

Keywords:
Social network
Opinion dynamics
Multi-agent systems
Distributed algorithms

ABSTRACT

In recent years, we have observed a significant trend towards filling the gap between social network analysis and control. This trend was enabled by the introduction of new mathematical models describing dynamics of social groups, the advancement in complex networks theory and multi-agent systems, and the development of modern computational tools for big data analysis. The aim of this tutorial is to highlight a novel chapter of control theory, dealing with applications to social systems, to the attention of the broad research community. This paper is the first part of the tutorial, and it is focused on the most classical models of social dynamics and on their relations to the recent achievements in multi-agent systems.

1. Introduction

The 20th century witnessed a crucial paradigm shift in social and behavioral sciences, which can be described as "moving from the description of social bodies to dynamic problems of changing group life" (Lewin, 1947). Unlike individualistic approaches, focused on individual choices and interests of social actors, the emerging theories dealt with structural properties of social groups, organizations and movements, focusing on social relations (or ties) among their members.

A breakthrough in the analysis of social groups was enabled by introducing a quantitative method for describing social relations, later called *sociometry* (Moreno, 1934; 1951). The pioneering work Moreno (1934) introduced an important graphical tool of *sociogram*, that is, "a graph that visualizes the underlying structure of a group and the position each individual has within it" (Moreno, 1934). The works Moreno (1934); 1951) also broadly used the term "network", meaning a group of individuals that are "bound together" by some long-term relationships. Later, the term *social network* was coined, which denotes a structure, constituted by *social actors* (individuals or organizations) and *social ties* among them. Sociometry has given birth to the interdisciplinary science of Social Network Analysis (SNA) (Freeman, 2004; Scott, 2000; Scott & Carrington, 2011; Wasserman & Faust, 1994), extensively using mathematical methods and algorithmic tools to study structural properties of social networks and social movements (Diani & McAdam, 2003). SNA is closely related to economics (Easley & Kleinberg, 2010; Jackson, 2008), political studies (Knoke, 1993), medicine and health care (O'Malley & Marsden, 2008). The development of SNA has inspired many important concepts of modern network theory (Newman, 2003a; Newman, Barabasi, & Watts, 2006; Strogatz, 2001) such as e.g. cliques and communities, centrality measures, small-world network, graph's density and clustering coefficient.

On a parallel line of research, Norbert Wiener introduced the general science of Cybernetics (Wiener, 1948; 1954) with the objective to unify systems, control and information theory. Wiener believed that this new science should become a powerful tool in studying social processes, arguing that "society can only be understood through a study of the messages and communication facilities which belong to it" (Wiener, 1954). Confirming Wiener's ideas, the development of social sciences in the 20th century has given birth to a new chapter of sociology, called "sociocybernetics" (Geyer, 1995) and led to the increasing comprehension that "the foundational problem of sociology is the coordination and control of social systems" (Friedkin, 2015). However, the realm of social systems has remained almost untouched by modern control theory

☆ The paper is supported by Russian Science Foundation (RSF) grant 14-29-00142 hosted by IPME RAS.
* Corresponding author.
E-mail address: anton.p.1982@ieee.org (A.V. Proskurnikov).

http://dx.doi.org/10.1016/j.arcontrol.2017.03.002
1367-5788/© 2017 Elsevier Ltd. All rights reserved.

in spite of the tremendous progress in control of complex large-scale systems (Annaswamy et al., 2017; Murray, 2003; Samad & Annaswamy, 2011).

The gap between the well-developed theory of SNA and control can be explained by the lack of mathematical models, describing social dynamics, and tools for quantitative analysis and numerical simulation of large-scale social groups. While many natural and engineered networks exhibit "spontaneous order" effects (Strogatz, 2003) (consensus, synchrony and other regular collective behaviors), social communities are often featured by highly "irregular" and sophisticated dynamics. Opinions of individuals and actions related to them often fail to reach consensus but rather exhibit persistent disagreement, e.g. clustering or cleavage (Friedkin, 2015). This requires to develop mathematical models that are sufficiently "rich" to capture the behavior of social actors but are also "simple" enough to be rigorously analyzed. Although various aspects of "social" and "group" dynamics have been studied in the sociological literature (Lewin, 1947; Sorokin, 1947), mathematical methods of SNA have focused on graph-theoretic properties of social networks, paying much less attention to dynamics over them. The relevant models have been mostly confined to very special processes, such as e.g. random walks, contagion and percolation (Newman, 2003a; Newman et al., 2006).

The recent years have witnessed an important tendency towards filling the gap between SNA and dynamical systems, giving rise to new theories of Dynamical Social Networks Analysis (DSNA) (Breiger, Carley, & Pattison, 2003) and *temporal* or *evolutionary* networks (Aggarwal & Subbian, 2014; Holme & Saramäki, 2013). Advancements in statistical physics have given rise to a new science of *sociodynamics* (Castellano, Fortunato, & Loreto, 2009; Weidlich, 2005), which stipulates analogies between social communities and physical systems. Besides theoretical methods for analysis of complex social processes, software tools for big data analysis have been developed, which enable an investigation of Online Social Networks such as Facebook and Twitter and dynamical processes over them (Arnaboldi, Passarella, Conti, & Dunbar, 2015).

Without any doubt, applications of multi-agent and networked control to social groups will become a key component of the emerging science on dynamic social networks. Although the models of social processes have been suggested in abundance (Acemoglu, Dahleh, Lobel, & Ozdaglar, 2011; Castellano et al., 2009; Friedkin, 2015; Mason, Conrey, & Smith, 2007; Xia, Wang, & Xuan, 2011), only a few of them have been rigorously analyzed from the system-theoretic viewpoint. Even less attention has been paid to their experimental validation, which requires to develop rigorous identification methods. A branch of control theory, addressing problems from social and behavioral sciences, is very young, and its contours are still blurred. Without aiming to provide a complete and exhaustive survey of this novel area at its dawn, this tutorial focuses on the most "mature" dynamic models and on the most influential mathematical results, related to them. These models and results are mainly concerned with *opinion formation* under social influence.

This paper, being the first part of the tutorial, introduces preliminary mathematical concepts and considers the four models of opinion evolution, introduced in 1950–1990s (but rigorously examined only recently): the models by French-DeGroot, Abelson, Friedkin–Johnsen and Taylor. We also discuss the relations between these models and modern multi-agent control, where some of them have been subsequently rediscovered. In the second part of the tutorial more advanced models of opinion evolution, the current trends and novel challenges for systems and control in social sciences will be considered.

The paper is organized as follows. Section 2 introduces some preliminary concepts, regarding multi-agent networks, graphs and matrices. In Section 3 we introduce the French-DeGroot model and discuss its relation to multi-agent consensus. Section 4 introduces a continuous-time counterpart of the French-DeGroot model, proposed by Abelson; in this section the Abelson diversity problem is also discussed. Sections 5 and 6 introduce, respectively, the Taylor and Friedkin–Johnsen models, describing opinion formation in presence of stubborn and prejudiced agents.

2. Opinions, agents, graphs and matrices

In this section, we discuss several important concepts, broadly used throughout the paper.

2.1. Approaches to opinion dynamics modeling

In this tutorial, we primarily deal with models of *opinion dynamics*. As discussed in Friedkin (2015), individuals' opinions stand for their *cognitive orientations* towards some objects (e.g. particular issues, events or other individuals), for instance, displayed *attitudes* (Abelson, 1964; Hunter, Danes, & Cohen, 1984; Kaplowitz & Fink, 1992) or subjective certainties of belief (Halpern, 1991). Mathematically, opinions are just scalar or vector quantities associated with social actors.

Up to now, system-theoretic studies on opinion dynamics have primarily focused on models with *real-valued* ("continuous") opinions, which can attain continuum of values and are treated as some quantities of interest, e.g. subjective probabilities (DeGroot, 1974; Li, Scaglione, Swami, & Zhao, 2013). These models obey systems of ordinary differential or difference equations and can be examined by conventional control-theoretic techniques. A discrete-valued scalar opinion is often associated with some action or decision taken by a social actor, e.g. to support some movement or abstain from it and to vote for or against a bill (Castellano et al., 2009; Clifford & Sudbury, 1973; Granovetter, 1978; Holley & Liggett, 1975; Sznajd-Weron & Sznajd, 2000; Weidlich, 1971; Yildiz, Ozdaglar, Acemoglu, Saberi, & Scaglione, 2013). A multidimensional discrete-valued opinion may be treated as a set of cultural traits (Axelrod, 1997). Analysis of discrete-valued opinion dynamics usually require techniques from advanced probability theory that are mainly beyond the scope of this tutorial.

Models of social dynamics can be divided into two major classes: macroscopic and microscopic models. Macroscopic models of opinion dynamics are similar in spirit to models of continuum mechanics, based on Euler's formalism; this approach to opinion modeling is also called Eulerian (Canuto, Fagnani, & Tilli, 2012; Mirtabatabaei, Jia, & Bullo, 2014) or statistical (Weidlich, 1971). Macroscopic models describe how the *distribution* of opinions (e.g. the vote preferences on some election or referendum) evolves over time. The statistical approach is typically used in "sociodynamics" (Weidlich, 2005) and evolutionary game theory (Easley & Kleinberg, 2010; Maynard Smith, 1982) (where the "opinions" of players stand for their strategies); some of macroscopic models date back to 1930–40s (Rashevsky, 1939; 1947).

Microscopic, or agent-based, models of opinion formation describes how opinions of individual social actors, henceforth called *agents*, evolve. There is an analogy between the microscopic approach, also called *aggregative* (Abelson, 1967), and the Lagrangian formalism in mechanics (Canuto et al., 2012). Unlike statistical models, adequate for very large groups (mathematically, the number of agents goes to infinity), agent-based models can describe both small-size and large-scale communities.

With the aim to provide a basic introduction to social dynamics modeling and analysis, this tutorial is confined to *agent-based* models with *real-valued* scalar and vector opinions, whereas other models are either skipped or mentioned briefly. All the models, considered in this paper, deal with an idealistic closed community,