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Review

From static output feedback to structured robust static output feedback: A survey

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ABSTRACT

This paper reviews the vast literature on static output feedback design for linear time-invariant systems including classical results and recent developments. In particular, we focus on static output feedback synthesis with performance specifications, structured static output feedback, and robustness. The paper provides a comprehensive review on existing design approaches including iterative linear matrix inequalities heuristics, linear matrix inequalities with rank constraints, methods with decoupled Lyapunov matrices, and non-Lyapunov-based approaches. We describe the main difficulties of dealing with static output feedback design and summarize the main features, advantages, and limitations of existing design methods.

1. Introduction

Static output feedback design is a theoretically challenging issue in control theory and it has attracted considerable attention due to its great importance in practice. However, so far, there has been no exact solution to this prominent problem which can guarantee the design of static output feedback or determine that such a feedback does not exist. The fact is that the problem is intrinsically a Bilinear Matrix Inequality (BMI) problem which is generally NP-hard (Toker & Ozbay, 1995); furthermore, it becomes non-smooth in the case of problem formulation in the space of the controller parameters (Toscano, 2013).

To solve the static output feedback design problem, well-known bilinear matrix inequality (BMI) solvers such as the commercial software package PENBMI (Henrion, Lofberg, Kocvara, & Stingl, 2005; Kocvara & Stingl, 2006) and the free open-source MATLAB toolbox PENLAB (Fiala, Kocvara, & Stingl, 2013) can be applied. The algorithms behind these solvers combines the ideas of the (exterior) penalty and (interior) barrier methods with the augmented Lagrangian approach (Kocvara & Stingl, 2003). These solvers can locally solve all kinds of BMI problems, including static output feedback. Since our aim is to survey dedicated static output feedback design methods, we no longer discuss these general BMI approaches. Note however that BMI solvers most often fail to provide a solution for the static output feedback BMI problems, and the choice of an initial guess is very crucial for these solvers.

The only survey dedicated to static output feedback has been conducted in Syrmos, Abdallah, Dorato, and Grigoriadis (1997). Since then, the past two decades have witnessed much theoretical progress on static output feedback design which has not been covered in that survey. A large amount of research has been carried out on the development of the static output feedback controllers according to Lyapunov theory via linear matrix inequality based (LMI-based) approaches (e.g. Agulhari, Oliveira, and Peres, 2012; Apkarian, Noll, and Tuan, 2003; Arzelier, Gryazina, Peaucelle, and Polyak, 2010; Arzelier and Peaucelle, 2002; Benton and Smith, 1998; Cao and Sun, 1998; Cao, Sun, and Mao, 1998; Dabboussi and Zrida, 2012; Dong and Yang, 2013; Du and Yang, 2008; Ebihara and Hagiwara, 2003; Ebihara, Tokuyama, and Hagiwara, 2004; Geromel, de Souza, and Skelton, 1998b; Ghaoui, Oustry, and Ait-Rami, 1997; Ghaoui and Balakrishnan, 1994; Grigoriadis and Beran, 2000; Grigoriadis and Skelton, 1996; Hassibi, How, and Boyd, 1999; Iwasaki, 1999; Iwasaki and Skelton, 1995b; Karimi and Sadabadi, 2013; Kim, Moon, and Kwon, 2007; Koroglu and Falcone, 2014; Lee, Lee, and Kwon, 2006; Leibfritz, 2001; Mehdi, Boukas, and Bachelier, 2004; Moreira, Oliveira, and Peres, 2011; Noll, Torki, and Apkarian, 2004; Peaucelle and Arzelier, 2001a; Sadabadi and Karimi, 2013a,b; 2015; Sadeghzadeh, 2014; Tran Dinh, Gumussoy, Michiels, and Diehl, 2012). Most of these methods present an iterative algorithm in which a set of LMIs

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is iteratively repeated until some certain termination criteria are met. In addition to the Lyapunov-based approaches, there exist non-Lyapunov-based static output feedback control strategies (see, e.g. Apkarian, 2013; Apkarian, Bompart, and Noll, 2007; Apkarian and Noll, 2006; Arzelier, Deaconu, Gumussoy, and Henrion, 2011; Burke, Henrion, and Overton, 2006b; Chesi, 2014; Gumussoy, Henrion, Millstone, and Overton, 2009; Gumussoy and Overton, 2008; Peretz, 2016).

The objective of this paper is to provide a comprehensive review on the existing static output feedback design methods. The main focus is on pure stabilizing static output feedback design with no other specification. But the paper also addresses the problem of structured feedback, simultaneous stabilization, multi-performance, and robust control design. All methods and approaches described in the survey are gathered in order to provide a comprehensive classification. All results have been reinterpreted and rewritten so as to fit a common notation/framework. The notation uniformization allows a simplified overview on the differences and resemblances of the results. It allows as well to provide direct extensions of the existing results for example using system duality. Due to the fact that fixed-order dynamic output-feedback can equivalently be transformed into static output feedback by introducing an augmented plant (Ghaoui et al., 1997), this survey paper can also be used for fixed/low-order control design problem.

The paper is organized as follows. Section 2 presents problem statement and main difficulties associated with stabilizing static output feedback design and its extensions to structured feedback, simultaneous stabilization, multi-performance, and robust control. The five sections that follow provide our classification of SOF design methods. Section 3 focuses on special cases where under specific structures of the open-loop system, the SOF problem becomes convex. Section 4 reviews the available literature on iterative LMI heuristics for the intrinsically BMI nature of SOF design. Section 5 covers the heuristics related to a reformulation of the SOF design as LMIs with rank constraints. While all the previous sections describe results build out of classical Lyapunov conditions, Section 6 is devoted to methods with decoupled Lyapunov matrices that have better characteristics with respect to robustness. Section 7 exposes alternative approaches which are non-Lyapunov-based. All the classes of results are analyzed in terms of their known or claimed numerical characteristics, well as in terms of their ability to address the structured feedback, simultaneous multi-performance, and robustness issues. The paper ends with global concluding remarks in Section 8.

The notation used in this paper is standard. In particular, matrices I and 0 are the identity matrix and the zero matrix of appropriate dimensions, respectively. The symbol $\star$ denotes symmetric blocks in block matrices. The symbols A^T, $\{A\}^{\mathcal{S}}$, $A^{\perp}$, $\|A\|_F$, and $A^{\frac{1}{2}}$ are respectively notations for the transpose of A, $\{A\}^{\mathcal{S}} = A + A^T$, the maximal rank perpendicularity such that $A^{\perp}A = 0$, Frobenius norm of A, and the unique nonnegative-definite square root of positive-definite matrix A. For symmetric matrices, $P > 0$ ($P < 0$) indicates the positive-definiteness (the negative-definiteness).

2. Problem formulation and main difficulties

2.1. Main SOF stabilization problem

Consider a linear time-invariant (LTI) continuous-time system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

and a static output feedback controller

$$u(t) = Ky(t) \tag{2}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^{n_i}$ in the control input, and $y \in \mathbb{R}^{n_o}$ is the output of the system. The state-space matrices A, B, C, and the control gain K are of appropriate dimensions. The closed-loop system is described as follows:

$$\dot{x}(t) = (A + BKC)x(t) \tag{3}$$

and its stability is equivalent to that of the dual system

$$\dot{x}_d(t) = (A + BKC)^T x_d(t). \tag{4}$$

Theorem 1. *The following statements are equivalent and prove that the static output feedback (2) stabilizes the system (1).*

(a) *The eigenvalues of $A + BKC$ are all in the left-half plane.*
(b) *There exists a symmetric matrix P satisfying the following matrix inequalities (Lyapunov inequalities for the primal system):*
$$P > 0, \quad \{P(A + BKC)\}^{\mathcal{S}} < 0 \tag{5}$$
(c) *There exists a symmetric matrix Q satisfying the following matrix inequalities (Lyapunov inequalities for the dual system):*
$$Q > 0, \quad \{(A + BKC)Q\}^{\mathcal{S}} < 0 \tag{6}$$
Moreover, $Q = P^{-1}$ holds to prove equivalence of the two last conditions.

The main difficulties associated with static output feedback design are as follows (Henrion, 2015):

- Non-differentiability: The performance objective related to the first statement (maximal real part of all eigenvalues) is a non-differential function of K. The spectral abscissa of the closed-loop state matrix $A + BKC$ is a continuous but non-Lipschitz function of K; thus, its gradient can be locally unbounded.
- Non-convexity: The stability conditions (5) or (6) are not convex in the unknowns due to the terms containing products of P and K and products of Q and K, respectively.

For concrete control system design, the problem formulation is scarcely limited to proving stability of the closed-loop. The actual problems to be solved include multi-objective and robustness specifications as well as structure constraints on the control gains. In this survey we shall not enter in all the details of how these specifications are formulated for each considered method, and most often they are not. We will rather give a general appreciation of the ability of the methods to address these specifications.

2.2. Structured SOF

Constraints on the control structure are mainly rooted in different sources. The first source comes from the well-known Internal Model Principle (IMP) (Francis & Wonham, 1976) which states that for tracking and disturbance rejection, the dynamics of persistently exciting references and/or disturbances must be replicated in the structure of the controller. Furthermore, the well-known proportional-integral (PI) and proportional-integral-derivative (PID) controllers, widely used in industrial control systems, inherently have a fixed structure. Finally, the last main source results from a need for decentralized or distributed control of large-scale interconnected systems due to cost, reliability issues, and limitations on communication links among the local controllers (Zecevic & Siljak, 2010). All these reasons highlight the paramount importance of structured control design.

Mathematically these structural constraints usually boil down to impose that some coefficients are zero in the K matrix and/or that some others are linearly dependent. More general non-linear constraints may also occur but for the present survey we shall assume that structure constraints are linear equality constraints of the type $L_s K R_s = C_s$ where there may be several triples of given matrices (L_s, R_s, C_s).