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Full Length Article

An Extended Zonotopic and Gaussian Kalman Filter (EZGKF) merging set-membership and stochastic paradigms: Toward non-linear filtering and fault detection☆

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ABSTRACT

A framework merging the set-membership and the stochastic paradigms is formalized and used to design an Extended Zonotopic and Gaussian Kalman Filter (EZGKF) dealing with the robust state estimation and the fault detection of uncertain discrete-time nonlinear systems. The so-called Set-membership and Gaussian Mergers (SGM) are introduced and particularized to Zonotopes (ZGM). They provide a constructive and computationally efficient solution to propagate random uncertainties with incompletely specified probability distributions combining set-based support enclosures and upper covariance matrix bounds formalized as matrix inequalities. Based on a full time-varying LPV enclosure featuring structured state matrix uncertainties, and given some confidence level expressed in probabilistic terms (maximal false alarm rate), a detection test is developed and shown to merge the usually mutually exclusive benefits granted by set-membership techniques (robustness to the worst-case within specified bounds, domain computations) and stochastic approaches (taking noise distribution into account, probabilistic evaluation of tests). A numerical example illustrates the state estimation capabilities of EZGKF and the improved tradeoff between the sensitivity to faults and the robustness to disturbances/noises.

1. Introduction

Relevant descriptions of disturbances and noises as well as efficient computational methods evaluating their propagation within system dynamics appear as a cornerstone of reliable state estimation and fault diagnosis. Indeed, not only a nominal trajectory but also a whole set of possible deviations from it must be characterized as accurately as possible for a faulty behavior to be almost surely detected in spite of uncertainties which are inherent to the modeling task. Either mainly based on some knowledge or some data, the models used for fault detection and diagnosis (Blanke, Kinnaert, Lunze, and Staroswiecki, 2003; Ding, 2008; Frank, 1990; Isermann, 2005) remain subject to such requirements to achieve a good tradeoff between the sensitivity to faults and the robustness to disturbances and noises. When dealing with uncertainties, two usually distinct paradigms can be used: the stochastic one and the set-membership (or bounded-error) one. Based on stochastic processes, Kalman filtering (Kalman, 1960; Maybeck, 1979) has been successfully used in a wide range of applications, including fault detection. Mainly based on Gaussian probability distributions (in spite of several kinds of extensions), it is often well suited to deal with measurement noises. However, the modeling of disturbances mostly related to some lack of knowledge about deterministic behaviors (e.g. load torque of a motor under incompletely specified operating conditions) is often more representative using bounded errors than Gaussian distributions. Indeed, such disturbances can successively vary arbitrarily, then temporarily remain constant but equal to unknown values, then vary again but differently, etc., and do not have any other stationary behavior than that of remaining within specified bounds, at least under fault-free operational conditions. Set-membership techniques, either based on ellipsoids (Kurzhanskiy & Varaiya, 2007; Maksarov & Norton, 2002; Schweppe, 1968), intervals (Jaulin, Kieffer, Didrit, & Walter, 2001; Moore, 1966; Raïssi, Efimov, & Zolghadri, 2012; Ramdani, Meslem, & Candau, 2009), polytopes or zonotopes (Combastel, 2003; 2015b; Kühn, 1998a; Le, Stoica, Alamo, Camacho, & Dumur, 2013; Puig, Saludes, & Quevedo, 2003) are well suited to deal with them. In this context, state bounding observers based on predictor/corrector approaches (so resembling

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Kalman filters) can be used (Combastel, 2003; Jaulin et al., 2001). However, contrary to stochastic Kalman filters which efficiently deal with random (measurement) noises, a bounded-error description of measurement noise often induces an unnecessary loss of precision reducing the sensitivity to faults.

This motivates the study of combined stochastic and set-theoretic uncertainties. This topic has not received much attention in the literature. However, Tjarnstrom and Garulli (2002) proposes a mixed approach to the identification of linear dynamic systems subject to an additive bounded white noise. Some works about interval Kalman filtering (Chen, Wang, and Leang Shieh, 1997; Xiong, Jauberthie, and Trave-Massuyes, 2013) already deal with Gaussian noises and unstructured interval state-space matrix uncertainties. The required interval matrix inversions may be addressed by approximations either leading to the loss of some solutions (Chen et al. 1997) or to some overestimation effects unless set inversion techniques are used Xiong et al. (2013). Efficient optimal Kalman gain computations for combined Gaussian and Ellipsoidal state estimation are proposed in Noack, Pfaff, and Hanebeck (2012). Based on a linear time invariant discrete-time system with Gaussian white noises as inputs, Shi, Chen, and Shi (2015) study set-valued Kalman filtering and its application to event-based state estimation. Sets of estimation means and ellipsoidal domains are used. Though very well suited to be combined with a Gaussian distribution, a single ellipsoid may hardly capture some refinements in the shape of the set of states consistent with generic interval-bounded disturbances. In Benavoli and Piga (2016), sets of probability measures are used in conjunction with polytopic bounding to reformulate set-membership estimation in a probabilistic setting, with an application to polynomial systems subject to bounded uncertainties. In Combastel (2015a), discrete-time LTV fault-free models simultaneously excited by bounded disturbances and Gaussian noises have been considered, and a computationally efficient solution merging Gaussian Kalman filtering and zonotopic state bounding for robust fault detection under noisy environment has been proposed.

Developing an Extended version of the Zonotopic and Gaussian Kalman Filter (ZGKF) first introduced in Combastel (2015a) is the main subject of this paper. Under mild assumptions, the proposed extension deals with the robust state estimation and the fault detection of uncertain nonlinear discrete-time models based on reliable LPV/LTV enclosures. To that purpose, an original framework merging set-membership and stochastic paradigms is formalized. Only upper bounds of covariance matrices are required to describe the Gaussian uncertainties involved in the so-called Set-membership and Gaussian Mergers (SGM) which are introduced here for the first time. The SGM are then particularized to zonotopes, so resulting in Zonotopic and Gaussian Mergers (ZGM), which permit to constructively and efficiently propagate random uncertainties under incompletely specified probability distributions. Prediction domains satisfying a given confidence level expressed in probabilistic terms can still be obtained, while affinely structured state matrix uncertainties are explicitly treated by the proposed Extended Zonotopic and Gaussian Kalman Filter (EZGKF). The algorithm is designed so as to combine the usually mutually exclusive benefits granted by set-membership techniques (robustness to the worst-case within specified bounds, domain computations) and stochastic approaches (taking noise distribution into account, probabilistic evaluation of tests e.g. false alarm rates). EZGKF can be used not only for robust state estimation but also for fault detection, as illustrated by a numerical example based on a discrete-time nonlinear prey-predator model.

The paper is organized as follows: after the preliminaries given in the Section 2, a framework merging the set-membership and the stochastic paradigms is formalized in the Section 3. The problem formulation follows in the Section 4. Based on the observer structure given in the Section 5, a multi-objective optimality criterion is proposed in the Section 6. Its purpose is to compute an optimal observer gain (Section 7) leading to the explicit EZGKF algorithm given in the Section 9 after dealing with structured state matrix uncertainties in the Section 8. Based on explicit prediction domains computed under a freely fixed confidence level, a fault detection test satisfying a requirement expressed as a probability of false alarms is proposed in the Section 10. A numerical example illustrating the use of EZGKF as a nonlinear filter performing not only the estimation of states but also the detection of faults is then reported in the Section 11.

2. Preliminaries

2.1. Probabilities: definitions and notations

Let $\mathfrak{P} = (\Omega, \Sigma, \mathcal{P})$ be a probability space, where Ω is a set of possible outcomes, $\Sigma \subset 2^{\Omega}$ (powerset of Ω) defines a collection of events, and $\mathcal{P}$ is a probability measure. Let $\mathbf{x}$ and $\mathbf{y}$ be two random real vectors defined on $\mathfrak{P}$. Boldface names denote random variables. The expectation of $\mathbf{x}$ is $\mathrm{E}[\mathbf{x}] = \int_{\Omega} \mathbf{x} d\mathcal{P}$. The operator E[.] is linear. The (cross)covariance between $\mathbf{x}$ and $\mathbf{y}$ is: $\mathrm{Cov}(\mathbf{x}, \mathbf{y}) = \mathrm{E}[(\mathbf{x} - \mathrm{E}[\mathbf{x}])(\mathbf{y} - \mathrm{E}[\mathbf{y}])^T] = \mathrm{E}[\mathbf{x}\mathbf{y}^T] - \mathrm{E}[\mathbf{x}]\mathrm{E}[\mathbf{y}]^T$. The covariance of $\mathbf{x}$ (or variance in the scalar case) is $\mathrm{Cov}(\mathbf{x}) = \mathrm{Cov}(\mathbf{x}, \mathbf{x})$. For continuous random vectors like $\mathbf{x}$, a probability density function (pdf), $\rho_{\mathbf{x}} : \mathbb{R}^n \to \mathbb{R}$ is such that $\forall \mathcal{D} \subset \mathbb{R}^n,\ \mathcal{P}(\mathbf{x} \in \mathcal{D}) = \int_{\mathcal{D}} \rho_{\mathbf{x}}(x) dx$, where $\mathcal{P}(\mathbf{x} \in \mathcal{D})$ is the probability that an outcome leads $\mathbf{x}$ to fall inside the domain $\mathcal{D}$. The support $\mathcal{S}_{\mathbf{x}}$ of $\mathbf{x}$ is the smallest closed set whose complement has probability zero. So, $\mathcal{P}(\mathbf{x} \in \mathcal{S}_{\mathbf{x}}) = 1$.

2.2. Gaussian distribution and prediction ellipsoids

Let $\mathbf{x} \sim \mathcal{N}(c, Q)$ refer to a random vector following a Gaussian (normal) probability distribution with center $c \in \mathbb{R}^n$ and covariance matrix $Q \in \mathbb{R}^{n \times n}$:

$$\rho_{\mathbf{x}}(x) = \frac{1}{\sqrt{(2\pi)^n \det(Q)}} \exp\left(-\frac{1}{2}(x - c)^T Q^{-1}(x - c)\right). \tag{1}$$

The support $\mathcal{S}_{\mathbf{x}} = \mathbb{R}^n$ is unbounded, but a prediction ellipsoid $(c, Q)_{\alpha}$ under a given confidence level can be defined as:

$$(c, Q)_{\alpha} = \{x \in \mathbb{R}^n,\ (x - c)^T Q^{-1}(x - c) \leq \chi_n^2(1 - \alpha)\}, \tag{2}$$

where $\chi_n^2(1 - \alpha) \in \mathbb{R}$ is the value taken for the probability $1 - \alpha$ by the quantile function of the chi-squared distribution with n degrees of freedom. The scalar parameter α can be interpreted as a probability of type I error (false alarm rate) when testing the membership of an outcome of the random vector $\mathbf{x}$ to the ellipsoidal set $(c, Q)_{\alpha}$:

$$\mathbf{x} \sim \mathcal{N}(c, Q) \Rightarrow \mathcal{P}(\mathbf{x} \in (c, Q)_{\alpha}) = 1 - \alpha. \tag{3}$$

2.3. Zonotopes

A zonotope $\langle c, R \rangle \subset \mathbb{R}^n$ with the center $c \in \mathbb{R}^n$ and the generator matrix $R \in \mathbb{R}^{n \times p}$ is a polytopic set defined as the linear image of the unit hypercube $[-1, +1]^p$ by R:

$$\langle c, R \rangle = \{c + Rs,\ \|s\|_{\infty} \leq 1\}. \tag{4}$$

The short notation $\langle R \rangle = \langle 0, R \rangle$ refers to a centered zonotope. Any permutation of the columns of R leaves it invariant. The Minkowski sum of two sets S_1 and S_2 is $S_1 \oplus S_2 = \{s_1 + s_2,\ (s_1, s_2) \in S_1 \times S_2\}$. The linear image of the set $S \subset \mathbb{R}^n$ by $L \in \mathbb{R}^{q \times n}$ is $L \odot S = \{Ls,\ s \in S\}$. Zonotopes form a class of polytopic sets implicitly represented by matrices and leading to efficient set computations. This class is closed under the Minkowski sum $\oplus$ (computed as a matrix concatenation) and the linear image $\odot$ (computed as a matrix

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