Annual Reviews in Control 000 (2016) 1–10

Contents lists available at ScienceDirect

Annual Reviews in Control

journal homepage: www.elsevier.com/locate/arcontrol

Review

Negative imaginary systems theory and applications

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ARTICLE INFO

Article history:
Received 29 May 2016
Revised 19 August 2016
Accepted 8 September 2016
Available online xxx

Keywords:
Robust control
Negative imaginary systems
Control of atomic force microscopes

ABSTRACT

This paper presents a survey of some of the main results in the theory of negative imaginary systems. The paper also presents some applications of negative imaginary systems theory in the design of robust controllers. In particular, the paper concentrates on the application of negative imaginary systems theory in the area of control of atomic force microscopes.

1. Introduction

The theory of negative imaginary systems is an emerging theory which is attracting increasing interest among control theory researchers; e.g., see (Benner & Voigt, 2013; Buscarino, Fortuna, & Frasca, 2016; Dey, Patra, & Siddhartha, 2016; Ferrante, Lanzon, & Ntogramatzidis, 2016; Ferrante & Ntogramatzidis., 2013; Lanzon & Petersen., 2008; Liu & Xiong, 2015; 2016; Opmeer, 2011; Petersen & Lanzon., 2010; Xiong, Lanzon, & Petersen, 2016b; Xiong, Petersen, & Lanzon., 2010; 2012). This theory is broadly applicable to problems of robust vibration control for flexible structures; e.g., see (Cai & Hagen., 2010b; Lanzon & Petersen., 2008; Mabrok, Kallapur, Petersen, & Lanzon, 2015; Petersen & Lanzon., 2010). Such flexible structures can be modelled by high order linear systems models with highly resonant dynamics (Halim & Moheimani, 2001; Pota, Moheimani, & Smith., 2002; Preumont, 2002). A particular problem in the control of such systems is the fact that unmodelled spillover dynamics can severely degrade control system performance or lead to instability if the controller is not designed to be robust against this type of uncertainty; e.g., see (Bhat & Miu., 1990; Dang, Lewis, Subbarao, & Stephanou., 2008). In addition, uncertainties in resonant frequencies and damping levels can cause similar problems of poor control system performance or instability. Negative imaginary systems theory provides a way analyzing robustness and designing robust controllers for such flexible structures in the case of collocated force actuators and position sensors; e.g., see (Bhikkaji, Moheimani, & Petersen., 2012; Cai & Hagen., 2010b; Mabrok, Kallapur, Petersen, & Lanzon, 2012; Mabrok, Haggag, & Petersen, 2016; Mabrok, Kallapur, Petersen, & Lanzon, 2014b; 2015; Mabrok, Lanzon, Kallapur, & Petersen, 2013; Mahmood, Moheimani, & Bhikkaji, 2011; McKelvey & Moheimani, 2005; Moheimani, Vautier, & Bhikkaji., 2006; Xiong, Lam, & Petersen, 2016a).

Negative imaginary systems arise in a wide variety of applications such as in the control of large space structures and the control of flexible dynamics in air vehicles. In addition, the theory of negative imaginary systems can be applied to areas of advanced technology involving nano-positioning. These features have led to an increasing number of researchers working on control applications to apply methods from negative imaginary systems theory; e.g., see (Abdullahi et al., 2015; Bhikkaji et al., 2012; Bhikkaji, Yong, Mahmood, & Moheimani, 2013; Cheekati & Bhikkaji, 2013; Das, Rehman, Pota, & Petersen, 2015c; Diaz, Pereira, & Reynolds, 2012; Fairbairn & Moheimani, 2012a; 2012b; Karvinen & Moheimani, 2013; 2014; Mahmood et al., 2011; Maroufi, Bazaei, & Moheimani, 2015; Pereira & Aphale, 2013; Rahman, Mamun, & Yao, 2015a; Rahman, Mamun, Yao, & Das, 2015b; Ruppert & Moheimani, 2015; Yue & Song, 2015). This emerging theory and its increasing adoption in control applications has motivated the current survey of some of the key elements of the theory and some important applications.

Nano-positioning such as in the control of atomic force microscopes (AFMs) is an important area of application for negative imaginary systems theory; e.g., see (Das, Pota, & Petersen, 2014; 2015a; 2015b; Das et al., 2015c). A schematic diagram describing the operation of an AFM is given in Fig. 1. In these applications, negative imaginary plant transfer functions commonly arise due to the use of piezoelectric force actuators and position sensors.

Negative imaginary systems theory aims to provide a general systems theory framework for the robust control of flexible structures. This theory allows for the design of controllers to increase the damping of the modes of a flexible structure. The controllers are also robust against uncertainty in the modal frequencies as well as unmodelled plant dynamics. An important special case of

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[1] The work of Ian R. Petersen was supported by the Australian Research Council under grants FL110100020 and DP160101121.
http://dx.doi.org/10.1016/j.arcontrol.2016.09.006
1367-5788/

Please cite this article as: I.R. Petersen, Negative imaginary systems theory and applications, Annual Reviews in Control (2016), http://dx.doi.org/10.1016/j.arcontrol.2016.09.006

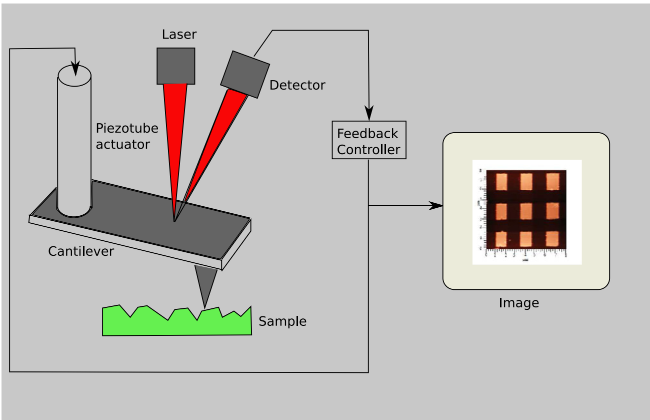

Fig. 1. Schematic Diagram of an Atomic Force Microscope.

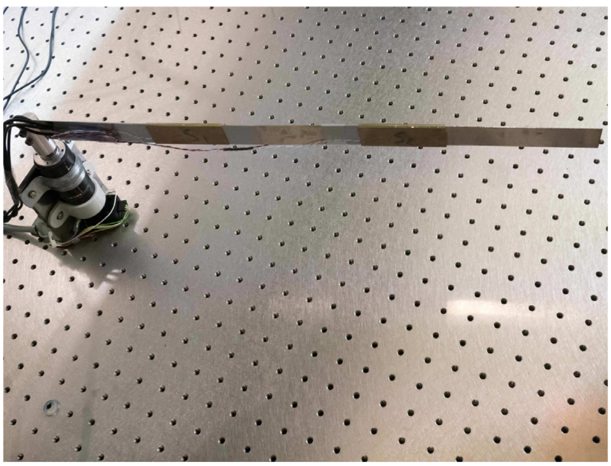

Fig. 2. Experimental setup for the control of a slewing flexible beam with piezoelectric actuators and sensors (at UNSW Canberra).

the control of flexible structures involves the use of piezoelectric actuators and sensors; see Fig. 2 which shows a laboratory experiment involving the control of a slewing flexible beam and the use of piezoelectric actuators and sensors together with a motor torque actuator and a shaft encoder position sensor; e.g., see (Mabrok, Kallapur, Petersen, & Lanzon, 2014a; Mabrok & Petersen, 2016).

Also note that the negative-imaginary theory of linear systems can be extended to nonlinear systems through the notion of counterclockwise input-output dynamics (Angeli., 2006; Cai & Hagen., 2010a).

2. Flexible structure modeling

In modeling an undamped flexible structure with collocated force actuators and position sensors, an infinite dimensional transfer function of the following form typically arises:

$$M(s) = \sum_{i=1}^{\infty} \frac{1}{s^2 + \kappa_i s + \omega_i^2} \psi_i \psi_i^{\mathrm{T}},$$

where, for all i, $\kappa_i > 0$, $\omega_i > 0$, and ψ_i is an $m \times 1$ vector; e.g., see (Preumont, 2002). Then, we consider the *Hermitian-imaginary part*

$$\Im_{\mathrm{H}}[M(\jmath\omega)] = -\frac{1}{2}\jmath(M(\jmath\omega) - M^*(\jmath\omega))$$

of this frequency response matrix $M(\jmath\omega)$. This quantity satisfies

$$\Im_{\mathrm{H}}[M(\jmath\omega)] = -\omega \sum_{i=1}^{\infty} \frac{\kappa_i}{\left(\omega_i^2 - \omega^2\right)^2 + \omega^2\kappa_i^2} \psi_i \psi_i^{\mathrm{T}} \leq 0$$

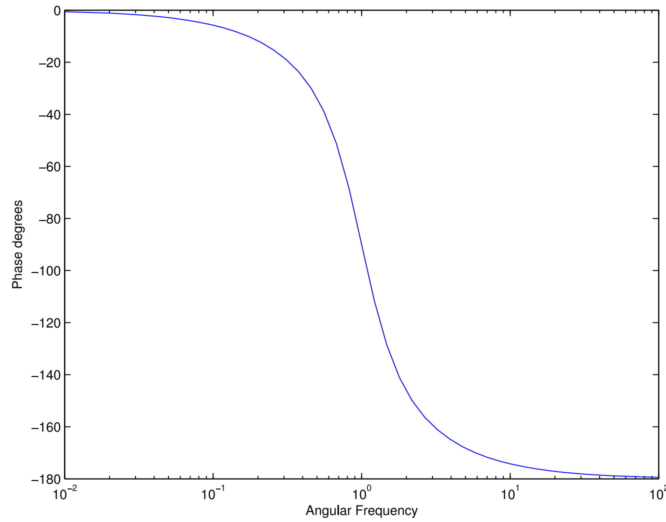

Fig. 3. Phase Bode plot of a typical negative imaginary transfer function.

for all $\omega \geq 0$. That is, $M(\jmath\omega)$ has a negative-semidefinite Hermitian-imaginary part for all $\omega \geq 0$. We thus refer to the transfer function matrix $M(s)$ as negative imaginary. A formal definition of the negative imaginary property will be given in the next section. Any flexible structure with collocated force actuators and position sensors will have a negative imaginary transfer function matrix; e.g., see (Petersen & Lanzon., 2010).

3. Negative imaginary systems

In this section, we present the formal definitions of the negative imaginary property and the strict negative imaginary property.

Definition 1 (Lanzon & Petersen., 2008; Xiong et al., 2010)**.** A square transfer function matrix $M(s)$ is *negative-imaginary (NI)* if the following conditions are satisfied:

1. $M(s)$ has no poles at the origin and in $\Re[s] > 0$;
2. $j[M(j\omega) - M^*(j\omega)] \geq 0$ for all $\omega \in (0, \infty)$ except values of ω where $j\omega$ is a pole of $M(s)$;
3. If $j\omega_0$, $\omega_0 \in (0, \infty)$, is a pole of $M(s)$, it is at most a simple pole, and the residue matrix $K_0 \triangleq \lim_{s \to j\omega_0} (s - j\omega_0) jM(s)$ is positive semidefinite Hermitian.

Remark 1. Note that this definition has been recently extended to allow for poles at the origin (Mabrok et al., 2014a). However, in this case, this leads to a more complicated form of the negative imaginary stability condition. Also, in Xiong, Petersen, and Lanzon. (2012) a lossless version of the negative imaginary property is given along with a corresponding negative imaginary lemma.

In the single-input single-output (SISO) case, a transfer function is negative imaginary if and only if it has no poles in open right half plane or the origin and its phase is in the interval $[-180, 0]$ degrees at all frequencies; e.g., see Fig. 3.

Consequently, the Nyquist plot of a SISO negative-imaginary transfer function lies below the real axis; e.g., see Fig. 4.

The following definition defines the strict negative imaginary property which is needed in the NI stability result.

Definition 2 (See (Lanzon & Petersen., 2008).)**.** A square real-rational proper transfer function matrix $M(s)$ is termed *strictly negative imaginary* (SNI) if

1. $M(s)$ has no poles in $\Re[s] \geq 0$;
2. $j[M(j\omega) - M^*(j\omega)] > 0$ for $\omega \in (0, \infty)$.