Automatica 87 (2018) 40–51

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Sparse and constrained stochastic predictive control for networked systems[☆]

Prabhat K. Mishra [a], Debasish Chatterjee [a], Daniel E. Quevedo [b]

[a] *Systems & Control Engineering, Indian Institute of Technology Bombay, India*
[b] *Department of Electrical Engineering (EIM-E), Paderborn University, Germany*

ARTICLE INFO

Article history:
Received 2 January 2017
Received in revised form 6 June 2017
Accepted 6 September 2017

Keywords:
Erasure channel
Stochastic predictive control
Networked system
Multiplicative noise
Unreliable channel
Sparsity

ABSTRACT

This article presents a novel class of control policies for networked control of Lyapunov-stable linear systems with bounded inputs. The control channel is assumed to have i.i.d. Bernoulli packet dropouts and the system is assumed to be affected by additive stochastic noise. Our proposed class of policies is affine in the past dropouts and saturated values of the past disturbances. We further consider a regularization term in a quadratic performance index to promote sparsity in control. We demonstrate how to augment the underlying optimization problem with a constant negative drift constraint to ensure mean-square boundedness of the closed-loop states, yielding a convex quadratic program to be solved periodically online. The states of the closed-loop plant under the receding horizon implementation of the proposed class of policies are mean square bounded for any positive bound on the control and any non-zero probability of successful transmission.

1. Introduction

An ever-increasing number of modern control technologies requires remote computation of control values that are then transmitted to the actuators over a network. Examples include heat, ventilation, and air-conditioning systems (HVAC) (Afram & Janabi-Sharifi, 2014; Kelman & Borrelli, 2011; Oldewurtel, Jones, Parisio, & Morari, 2014; Parisio, Varagnolo, Risberg, Pattarello, Molinari, & Johansson, 2013) and cloud-aided vehicle control systems (Alessandretti, Aguiar, & Jones, 2015; Li, Kolmanovsky, Atkins, Lu, & Filev, 2015; Li, Kolmanovsky, Atkins, Lu, Filev, & Michelini, 2014; Ogitsu & Omae, 2015). In all such systems, a crucial role is played by the transmission channel and the communication protocol employed for the transmission of control commands. Due to fading and interference, the transmitted control commands may be delayed or lost, thereby affecting both qualitative and quantitative properties of the system. Since in networked systems rate limited channels are shared among various devices, sparse controls are also desirable and tractability is essential for the implementation. Moreover, from an operational stand point, in almost all practical applications there are hard constraints on the controls, and standard control design methods do not apply directly. Furthermore, since stability is one of the most desirable features, it is important to guarantee stability in the context of imperfect communication channel, stochastic noise and bounded controls. This article proposes a sparse, computationally tractable, constrained and stabilizing networked control method for stochastic systems based on predictive control techniques.

Predictive control techniques provide tractable solutions to constrained control problems by minimizing some suitably chosen performance index over a finite temporal horizon via an iterative procedure. Based on the choice of the performance index, in context of stochastic systems, these techniques are classified as certainty-equivalent (CE) and stochastic; see Fig. 1. CE approaches do not take advantage of the available statistics of the uncertainties; here only the nominal plant model is considered and the control selection procedure is over open loop input sequences. CE techniques are typically implemented over networks with help of a buffer and a smart actuator; the technique is commonly known as packetized predictive control (PPC) (Quevedo & Nešić, 2012). In PPC, the time stamped sequences containing the future values of the control are transmitted at each time instant, and the successfully received sequences are saved in a buffer at the actuator. In case of dropouts, the most recent value of the control taken from the buffer is applied to the plant. PPC, in this way, compensates the effect of dropouts, but the controller is itself deterministic, i.e., the performance index does not incorporate the effect of unreliable communication and additive process noise. Thus, it is

☆ D. Chatterjee was supported in part by the grant 12IRCCSG005 from IRCC, IIT Bombay. The material in this paper was partially presented at 10th IFAC Symposium on Nonlinear Control Systems, August 23–25, 2016, Monterey, CA, USA. This paper was recommended for publication in revised form by Associate Editor Riccardo Scattolini under the direction of Editor Ian R. Petersen. Authors are thankful to Prof. Masaaki Nagahara and Prof. D. Manjunath for helpful discussions.

E-mail addresses: prabhat@sc.iitb.ac.in (P.K. Mishra), dchatter@iitb.ac.in (D. Chatterjee), dquevedo@ieee.org (D.E. Quevedo).

https://doi.org/10.1016/j.automatica.2017.09.013
0005-1098/

quite intuitive, and has been argued in Quevedo, Mishra, Findeisen, and Chatterjee (2015) with help of numerical experiments, that a suitably chosen stochastic performance index compensating the effect of uncertainty propagation can outperform PPC.

Stochastic approaches incorporate the effects of uncertainties in predicted performance by considering the expected value of the cost per sample path in stochastic systems, and controlling with the help of policies as opposed to open-loop sequences. Typically, such policies are parametrized in some convenient way, and the cost is minimized over the associated set of decision variables.[1] It is well known that feedback of past additive disturbances leads to convex problems, whereas the state feedback approach leads to non-convexity in the set of decision variables (Goulart, Kerrigan, & Maciejowski, 2006). In order to obtain a convex set of feasible decision variables, disturbance feedback approaches have been studied extensively (Ben-Tal, Goryashko, Guslitzer, & Nemirovski, 2004; Garstka & Wets, 1974; Goulart et al., 2006; Guslitser, 2002; Löfberg, 2003; Van Hessem & Bosgra, 2002). To satisfy hard bounds on the control, saturated values of past disturbances are used in Hokayem, Chatterjee, and Lygeros (2009). This saturated disturbance feedback policy is applied to networked systems with sufficient control authority in Chatterjee, Amin, Hokayem, Lygeros, and Sastry (2010) and was later generalized to any positive bound on the control in our work (Mishra, Chatterjee, & Quevedo, 2016). We demonstrated in our recent conference contribution (Mishra, Quevedo, & Chatterjee, 2016) that in the absence of additive noise, the parametrization relative to past dropouts also leads to convex problems and outperforms approaches that merely minimize over open loop input sequences. This suggests that a parametrization relative to *both* past dropouts and past disturbances leads to an improved class of feedback policies.

Stochastic predictive control for networked systems is based on a suitable choice of the cost function and the class of control policies, a protocol to decide what the controller will transmit and what the actuator will do. In our previous contributions (Mishra et al., 2016; Mishra, Chatterjee, & Quevedo, 2016; Quevedo et al., 2015), we systematically developed a class of stochastic predictive control techniques for networked systems. We proposed transmission protocols in Mishra et al. (2016) to answer what the controller should transmit and what the actuator should do under the class of feedback policies adopted from (Hokayem et al., 2009). Stochastic approaches proposed so far (Hokayem et al., 2009; Mishra et al., 2016; Mishra et al., 2016; Mishra, Chatterjee, & Quevedo, 2017; Quevedo et al., 2015) neither consider communication effects in feedback policies nor generate sparse control vectors. In this article, we propose an affine dropout and saturated disturbance feedback policy for stochastic systems controlled over unreliable and rate limited channels. Here, going beyond our earlier works, we focus on control-communication co-design by employing a sparsity promoting optimization program. We utilize the ideas of compressed sensing (Elad, 2010) as in Bhattacharya and Başar (2011); Nagahara, Quevedo, and Ostergaard (2014). In Bhattacharya and Başar (2011), a sparsity based feedback system for the nominal plant model is presented and in Nagahara et al. (2014) sparse controls are designed for networked systems in absence of process noise, by exploiting a sparsity promoting regularization term, namely the ℓ_1-norm of the control vector. In the present work, we employ the mixed induced ℓ_1/ℓ_∞ norm for the regularization term in presence of the feedback policy. To the best of our knowledge, this is the first work where the effects of both the process noise and the dropouts are considered in a feedback policy, sparsity in control is promoted, and stochastic stability is guaranteed.

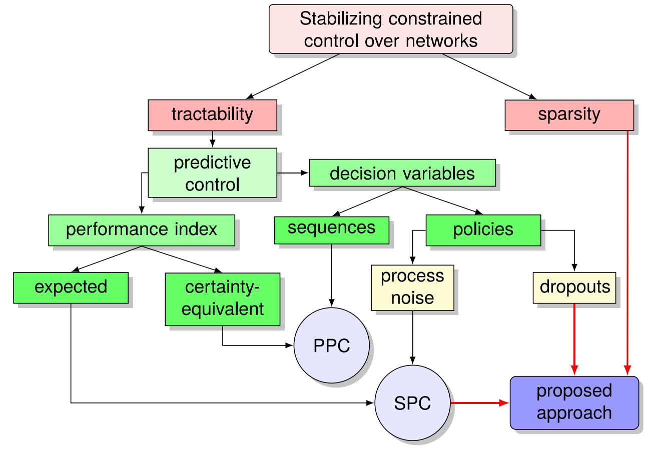

Fig. 1. The approach proposed in the present article extends the results of stochastic predictive control (SPC) as proposed in Mishra et al. (2016) by incorporating communication imperfection models and protocols explicitly into the controller design. In particular, past dropouts are considered in the feedback policy to formulate an sparsity-promoting optimization program. Here, PPC stands for packetized predictive control as described in Quevedo and Nešić (2012).

Our main contributions in this article are as follows:

- We propose a policy affine in past dropouts and saturated disturbances for a finite horizon optimal control problem. The resulting problem is shown to be convex and therefore numerically tractable.
- Stability constraints are incorporated into the underlying optimal control problem. For *any* positive bound on the control and for *any* non-zero successful transmission probability, these constraints ensure mean square boundedness of the system states for the largest class of linear systems with disturbances that are currently known to be stabilizable with bounded controls.
- We introduce a regularization term in the objective function of the underlying optimal control problem to promote sparsity in time of the applied controls. Sparsity of the control commands in time is useful to reduce communication through shared channels, and increases the relaxation time for the actuator.
- The objective function design is capable to incorporate the effects of communication channel and also control policy. This takes into account all sources of randomness in the considered networked control system, see Fig. 2.

This article exposes as follows: In Section 2 we establish notation and definitions of the plant and its properties. In Section 3 we present elementary aspects of constrained optimal control problems for stochastic systems. Our proposed class of feedback policies is presented in Section 4. We have introduced the sparsity promoting optimal control problem in Section 5. Implementation of the stabilizing feedback policy over networks is discussed in Section 6 and the computational aspects in Section 7. In Section 8, we discuss stability issues and present stability constraints for the proposed control algorithm. We validate our results with help of numerical experiments in Section 9. We conclude in Section 10. The proofs of our main results are documented in appendix Appendix A in consolidated manner.

Our notations are standard. We let $\mathbb{R}$ denote the real numbers and $\mathbb{N}$ denote the positive integers. The set of the non-negative reals and non-negative integers is denoted by $\mathbb{R}_{\geqslant 0}$ and $\mathbb{N}_0$, respectively. For any sequence $(s_n)_{n\in\mathbb{N}_0}$ taking values in some Euclidean

[1] Notice that the optimization over open-loop input sequences does not give optimal performance in the presence of uncertainties (Kumar & Varaiya, 1986 pp. 13–14), and therefore, optimization over feedback policies is preferred for stochastic systems.