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Adaptive control of linear 2×2 hyperbolic systems ☆

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ABSTRACT

We design two closely related state feedback adaptive control laws for stabilization of a class of 2×2 linear hyperbolic system of partial differential equations (PDEs) with constant but uncertain in-domain and boundary parameters. One control law uses an identifier, while the other is based on swapping design. We establish boundedness of all signals in the closed loop system, pointwise in space and time, and convergence of the system states to zero pointwise in space. The theory is demonstrated in simulations.

1. Introduction

1.1. Background

We will in this paper consider systems on the form of 2×2 linear hyperbolic partial differential equations (PDEs), which can be used to model for instance traffic flow (Goatin, 2006) and pressure and flow profiles in oil wells (Landet, Pavlov, & Aamo, 2013). Since equations of this type can be used to model a vast range of different physical systems, extensive research regarding control of this kind of systems have been performed, and we list control Lyapunov functions (Coron, d'Andréa Novel, & Bastin, 2007), Riemann invariants (Greenberg & Tsien, 1984) and frequency approaches (Litrico & Fromion, 2006) to name a few.

The pioneering backstepping approach presented in Liu (2003) for stabilization of partial differential equations of the parabolic type, has in recent years shown to be quite useful and a general framework for analysis of PDEs. The key ingredient of this approach is the introduction of an invertible Volterra-like transformation that maps the system to be investigated into an auxiliary system designed to possess some desirable stability properties. Due to the invertibility of the transformation, the stability properties of the two systems are the same.

The first use of backstepping to hyperbolic systems was presented in Krstić and Smyshlyaev (2008b), where among other applications, hyperbolic PDEs were used to model actuator and sensor delays in ordinary differential equations. Extensions of the backstepping technique to second order hyperbolic systems were presented in Smyshlyaev, Cerpa, and Krstić (2010), and in Vazquez, Krstić, and Coron (2011) to 2×2 coupled linear hyperbolic PDEs. Explicit non-adaptive controllers for a subclass of the systems covered in Vazquez et al. (2011) were also offered in Vazquez and Krstić (2014).

Adaptive stabilization of PDEs with unknown system parameters is a field that is well-established in the case of parabolic PDEs, with contributions like Krstić & Smyshlyaev (2008a); Smyshlyaev & Krstić (2007a, b, 2010). Material regarding adaptive control of hyperbolic PDEs, however, is currently limited. The first result was presented in Bernard and Krstić (2014), where an adaptive output feedback control law was derived for a single hyperbolic partial-integro differential equation with non-local source terms, while a subproblem of this was presented in Xu and Liu (2016) offering a full-state feedback solution. Recently, state feedback stabilization of coupled 2×2 linear hyperbolic systems of PDEs with uncertain in-domain coefficients was solved in Anfinsen and Aamo (2016a, b) using an identifier and swapping design, respectively. Boundedness and square integrability in the L_2-sense of the states were established, while the important result of convergence of the states to zero was not established. In the present paper, boundedness, square integrability and convergence to zero of system states pointwise in space are provided, thereby completing the missing aspects of Anfinsen and Aamo (2016a, b). Another minor extension is provided by considering the boundary parameters unknown in addition to the in-domain coefficients considered in Anfinsen and Aamo (2016a, b). A significant drawback of the result, limiting its practical value, is the need for full state measurements. Full state measurements are rarely available in practice, however, for the particular problem motivating the present work, they can be considered available in an approximate sense. When drilling oil wells,

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it is important to control pressure accurately in the well. The main obstacle to accurate modelling of the flow dynamics in the well is the uncertainty of friction parameters, which appear as coupling terms in the domain of the hyperbolic PDE. Emerging technology, referred to as wired pipe, allows for distributed sensing along the drill-pipe throughout the well. Sensors can be installed at every pipe connection, about 30 m apart, thereby providing an approximate measurement of the distributed state of the PDE. That being said, solving the output feedback problem is the ultimate goal, and is a topic of our current research.

1.2. Paper structure

In Section 2, we formally pose the control problem to be investigated. An adaptive control law based on an identifier is presented in Section 3. The control law is formally stated as Theorem 4. Then, in Section 4, another control law based on swapping design is presented, and the control law is formally stated as Theorem 7. Boundedness and square integrability of all states in the closed loop system in the L_2-sense are proved for both controllers, and pointwise boundedness, square integrability and convergence to zero of the system states are also proved. The performance of the controllers is demonstrated in simulations in Section 5, while Section 6 offers some concluding remarks, and lists some pros and cons regarding the two proposed controllers.

1.3. Notation

For a time-varying, real signal $f(t)$, the following vector spaces are used

$$f \in \mathcal{L}_p \Leftrightarrow \left(\int_0^\infty |f(t)|^p dt\right)^{\frac{1}{p}} < \infty \tag{1}$$

for $p \geq 1$ with the particular case

$$f \in \mathcal{L}_\infty \Leftrightarrow \sup_{t\geq 0}|f(t)| < \infty. \tag{2}$$

For the (possibly time-varying) vector signal $u(x)$ defined for $0 \leq x \leq 1$, we introduce the following integral operator

$$I_a[u] = \int_0^1 e^{ax}u(x)dx \tag{3}$$

with the derived norm

$$\|u\|_a^2 = I_a\left[u^T u\right] = \int_0^1 e^{ax}u^T(x)u(x)dx. \tag{4}$$

The operator (3) is linear and has the property

$$2I_a[uu_x] = e^a u^2(1) - u^2(0) - a\|u\|_a^2. \tag{5}$$

The norm $\|u\|_a$ is equivalent to the standard L_2 norm, in the sense that there exist positive constants k_1, k_2 so that

$$k_1\|u\|_a \leq \|u\| \leq k_2\|u\|_a, \tag{6}$$

and also that $\|u\| = \|u\|_0$. Moreover, for the sum of norms of u and v the shorthand notation

$$\|u, v\| = \|u\| + \|v\| \tag{7}$$

is used. Lastly, we will in subsequent sections often omit writing the argument in time, so that e.g. $u(x) = u(x, t)$ and $\|z\| = \|z(t)\|$.

2. Problem description

We consider systems on the form of 2×2 linear hyperbolic partial differential equations with constant in-domain coefficients. These type of systems were also investigated (Vazquez & Krstić, 2014), and are on the form

$$u_t(x,t) + \lambda u_x(x,t) = c_1 u(x,t) + c_2 v(x,t) \tag{8a}$$
$$v_t(x,t) - \mu v_x(x,t) = c_3 u(x,t) + c_4 v(x,t) \tag{8b}$$
$$u(0,t) = qv(0,t) \tag{8c}$$
$$v(1,t) = U(t) \tag{8d}$$

defined for $0 \leq x \leq 1$, $t \geq 0$, where u, v are the system states, and

$$0 < \lambda \in \mathbb{R}, \quad 0 < \mu \in \mathbb{R} \tag{9}$$

are known transport speeds while the coefficients

$$c_1, c_2, c_3, c_4, q \in \mathbb{R} \tag{10}$$

are unknown. However, we assume we have some bounds on c_i, $i = 1 \ldots 4$ and q. That is, we are in possession of some positive constants $\bar{c}_i$, $i = 1 \ldots 4$ and $\bar{q}$ so that

$$|c_i| \leq \bar{c}_i, \ i = 1 \ldots 4, \qquad |q| \leq \bar{q}. \tag{11}$$

These assumptions merely accommodate the use of the projection operator (see Appendix A for the definition and properties) to limit the parameter estimates, and do not restrict the class of systems (8) considered since the bounds are arbitrary. Finally, we assume the initial states $u(x,0) = u_0(x)$, $v(x,0) = v_0(x)$ satisfy

$$u_0, v_0 \in L_2. \tag{12}$$

The goal is to design a state feedback adaptive control law U that achieves regulation of the system states u and v to zero pointwise in space and time. Moreover, all additional signals should be bounded.

3. Adaptive control using an identifier

3.1. Introduction

In identifier-based design, a dynamical system – referred to as an identifier – is introduced. The identifier is usually a copy of the system dynamics with certain injection gains added for the purpose of making the adaptive laws integrable. Lyapunov theory is then used to derive adaptive laws, and also prove that the error between the system states and identifier states is bounded. The backstepping technique is used for controller design and to map the identifier into a target system for which stability analysis is easier. Boundedness of the identifier is then proved using the target system. Due to invertibility of the backstepping transform and the estimation error also being bounded, the original system states are bounded as well. An identifier is sometimes termed an observer, although its purpose is parameter estimation and not state estimation.

3.2. Identification using an identifier

Consider the identifier

$$\partial_t \hat{u}_1(x) + \lambda \partial_x \hat{u}_1(x) = \varpi^T(x)\hat{b}_1 + \rho e_1(x)\|\varpi\|^2 \tag{13a}$$
$$\partial_t \hat{v}_1(x) - \mu \partial_x \hat{v}_1(x) = \varpi^T(x)\hat{b}_2 + \rho \epsilon_1(x)\|\varpi\|^2 \tag{13b}$$
$$\hat{u}_1(0) = \frac{\hat{q}v(0) + u(0)v^2(0)}{1 + v^2(0)} \tag{13c}$$
$$\hat{v}_1(1) = U \tag{13d}$$

for some design gain $\rho > 0$, and where

$$e_1(x) = u(x) - \hat{u}_1(x) \tag{14a}$$
$$\epsilon_1(x) = v(x) - \hat{v}_1(x) \tag{14b}$$