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Negotiated distributed estimation with guaranteed performance for bandwidth-limited situations☆

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ABSTRACT

This paper proposes a guaranteed distributed estimator for large-scale, state-coupled plants. Each agent must find appropriate sets, described by zonotopes, to contain the actual state of the plant and reduce the estimation uncertainties. To do so, the agents collaborate by sending information through a network in which bandwidth restrictions need to be taken into account. An estimation algorithm is proposed, where the amount of information transmitted is negotiated through an iterative procedure that trades off communication burden and estimation performance. The negotiation process between the agents is similar to a non-cooperative game, which ends when the Nash equilibrium is reached. The algorithm can be tuned to weight estimation performance, communication costs, and discrepancies in the amount of information transmitted by neighboring agents. The estimation structure is tested by simulation on a plant consisting of a set of inverted pendulums coupled by springs.

1. Introduction

Since the emergence of the Kalman and Luenberger observers in the 1960s, state estimation has been subject of both theoretical and application-oriented research. The estimation of the state of a system, directly measurable only in rare occasions, allows us to monitor the evolution of important variables, tele-operate systems, or apply state feedback controllers. Nowadays, the development of wireless networks and embedded devices is transforming the architecture of estimation systems from point-to-point to distributed estimators implemented in smart agents with processing and sensing capabilities. The potentialities of these new systems are huge, ranging from traffic control or smart-grids, to human and robots collaborative applications.

This propitious context has favored fruitful research in agent-based distributed estimation, resulting in novel promising contributions to the topic. Distributed Kalman Filtering (DKF) approaches have shown themselves as efficient and flexible adaptations of the Kalman filter (Alriksson & Rantzer, 2006; Khan & Moura, 2008; Ribeiro, Giannakis, & Roumeliotis, 2006; Song, Zhu, Zhou, & You, 2007), and it has been combined with consensus (Battistelli & Chisci, 2016; Olfati-Saber, 2009), or diffusion strategies (Cattivelli & Sayed, 2010). Furthermore, consensus-based techniques have been also successfully applied to the same problem, taking explicitly into account time delays (Millán, Orihuela, Vivas, & Rubio, 2012), noisy links (Schizas, Ribeiro, & Giannakis, 2008), communication faults (Stanković, Stanković, & Stipanović, 2009), asynchronous communication (Millán, Orihuela, Jurado, Vivas, & Rubio, 2015), or reduced-order observers (Orihuela, Millán, Vivas, & Rubio, 2013; Park & Martins, 2017). Other approaches to the problem have considered H_∞ filtering (Shen, Wang, & Hung, 2010; Ugrinovskii, 2013), or moving-horizon techniques (Farina, Ferrari-Trecate, & Scattolini, 2010).

A different approach is the set-membership (SM) estimation. Unlike probabilistic approaches, SM techniques aim to confine the state within a certain set, which has important applications such as collision avoidance in robotics, critical systems where certain states must not exceed some given thresholds, or fault-detection algorithms, identifying sensors or actuators faults. To characterize these sets, different authors have resorted to variants of ellipses (Bertsekas & Rhodes, 1971), interval analysis tools (Combastel, 2013; Mazenc & Bernard, 2011; Raïssi, Ramdani, & Candau, 2004), or zonotopes (Alamo, Bravo, & Camacho, 2005; Combastel, 2015; Le, Stoica, Alamo, Camacho, & Didier, 2013). In the latter approach,

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derived from parallelotopic descriptions (Chisci, Garulli, & Zappa, 1996; Rapaport & Gouzé, 2003), a set is represented in terms of transformed hypercubes characterized by a vector, that defines its center, and by a matrix, that implicitly determines its shape and bounds.

The SM approach for state estimation based on zonotopes is very suitable for distributed implementations. On the one hand, the fact that these sets can be represented in terms of vectors and matrices eases transmission of information between agents. On the other hand, basic operations with zonotopes are reduced to matrix calculations, simple enough to be carried out in distributed embedded systems with limited computation capabilities. Compared to the ellipsoidal characterization, in which a set is determined by a square matrix, zonotopes produce more accurate descriptions.

Despite these advantages, the literature concerned with distributed estimation based on zonotopes is very scarce, being limited to some preliminary results of the authors of this paper (Garcia, Millan, Orihuela, Ortega, & Rubio, 2015), and also the recent works (Riverso, Rubini, & Ferrari-Trecate, 2015; Song, Yu, & Zhang, 2013). In Riverso et al. (2015), a distributed SM estimator is proposed based on zonotopes and on the concept of practical robust positive invariance. However, the convergence of state estimates is only guaranteed in the absence of disturbances and measurement noises, and therefore the approach suffers from conservativeness. Furthermore, the design of the observer gains implies centralized computations, which hinders the implementation in distributed systems. The work (Song et al., 2013) develops a distributed SM observer for limited communication data rate that deepens in the impact of logarithmic quantization on the estimation performance.

This paper proposes a novel set-membership, agent-based, distributed estimation technique for large-scale plants composed by a number of coupled subsystems. Each agent measures local outputs and communicates with its neighbors in order to improve the estimation performance. First, a guaranteed estimation algorithm based on zonotopes is developed, providing a design of the observer gains to minimize the estimation uncertainties. This distributed estimator can be viewed as an extension of the zonotopic Kalman filters (Combastel, 2015) to a distributed setting. After that, the situation with constrained network bandwidth is considered, and a negotiation method is proposed to reduce the amount of information to be sent at each sampling time. This negotiation considers the trade-off between communication burden and estimation performance, and its solution tends to the Nash equilibrium of a non-cooperative game (Basar & Olsder, 1999). Finally, conditions for the existence and stability of the Nash equilibrium are provided, so that the convergence of the negotiation can be ensured.

The paper is organized as follows. Section 2 presents some notation and preliminaries. The problem under consideration and the objectives pursued are detailed in Section 3. The proposed distributed observer is presented in Section 4. Section 5 develops the negotiation method when constrained communication is considered. Some illustrative examples are given in Section 6. Finally, conclusions are drawn in Section 7.

2. Notation and preliminaries

Let $R \in \mathbb{R}^{n\times p}$. Then, $\|R\|_F = \sqrt{tr(R^T R)}$ is the Frobenius norm of R. The maximum singular value of R is denoted by $\sigma_{\max}\{R\}$. Given matrices A, B of appropriate dimensions, operator cat$\{A, B\}$ implies the concatenation of the matrices, that is, cat$\{A, B\} = [A\ B]$.

A zonotope $\mathcal{X}$, denoted in this paper with calligraphic, capital letters, is a centrally symmetric, convex set determined by its center $c \in \mathbb{R}^n$, and by a matrix $H \in \mathbb{R}^{n\times p}$: $\mathcal{X} = \langle c, H\rangle = \left\{c + \sum_{i=1}^{p} \varsigma_i h_i : \forall i\ |\varsigma_i| \le 1\right\}$, where $h_i \in \mathbb{R}^n$ (columns of H) are called *generator vectors*. The *order* of a zonotope is given by the number of generator vectors, its *F-radius* is the Frobenius norm of H, and its *covariation* is defined as $P_{\mathcal{X}} = HH^T$ (see Combastel, 2015).

Let $\mathcal{X} = \langle c_x, H_x\rangle$ and $\mathcal{Y} = \langle c_y, H_y\rangle$ be two zonotopes, and let R be a matrix of appropriate dimensions. A linear transformation of a zonotope is given by $R\mathcal{X} = \langle Rc_x, RH_x\rangle$, and the Minkowski sum of two zonotopes is obtained as $\mathcal{X} \oplus \mathcal{Y} = \langle c_x + c_y,\ \text{cat}\{H_x, H_y\}\rangle$. Given a matrix A and any vectors such that $x \in \mathcal{X}$ and $w \in \mathcal{W}$, it holds that $y := Ax + w \in A\mathcal{X} \oplus \mathcal{W}$. The operator $\underset{q}{\text{red}}(\mathcal{X})$, defined as in Combastel (2015), is an order reduction of the zonotope $\mathcal{X}$ in such a way that $\mathcal{X} \subseteq \underset{q}{\text{red}}(\mathcal{X})$, and the order of $\underset{q}{\text{red}}(\mathcal{X})$ is q.

In this paper, the agents are considered to be connected by means of a communication network whose topology is defined by an undirected, connected graph, with vertices $V = \{1, 2, \ldots, b\}$ (agents) , and edges $E \subseteq V \times V$ (communication links). The set of agents to which agent i is directly connected is named *the neighborhood of* i, and is denoted by $N_i := \{j : (i, j) \in E\}$.

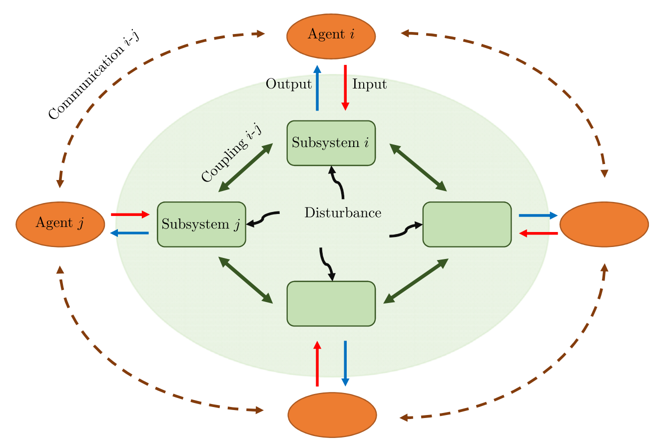

Fig. 1. Diagram of the large-scale plant with the distributed network of agents.

3. Problem statement

In this paper, we consider a large-scale plant comprised of a set of linear subsystems internally coupled as:

$$x_i(k+1) = A_{ii}x_i(k) + \sum_{j\in N_i} A_{ij}x_j(k) + B_i u_i(k) + D_i w_i(k), \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$ is the state of subsystem i $(i = 1, 2 \ldots, b)$, $u_i \in \mathbb{R}^{n_{u_i}}$ is a local control signal, $w_i \in \mathbb{R}^{n_{w_i}}$ represents disturbances or unmodeled dynamics, and A_{ii}, A_{ij}, B_i, D_i are matrices of appropriate dimensions (see Fig. 1).

The state of each subsystem is not directly accessible. Instead, the following outputs can be measured:

$$y_i(k) = C_i x_i(k) + v_i(k), \in \mathbb{R}^{n_{y_i}}, \tag{2}$$

where $v_i \in \mathbb{R}^{n_{y_i}}$ are measurement noises.

The following assumptions are considered:

Observability/detectability: The local pairs (A_{ii}, C_i) are detectable, this meaning that, in the absence of couplings, the state of each subsystem can be estimated using only local information.

Controllability: Control actions $u_i(k)$ can be computed such that the whole system can be stabilized. Since the control problem is out of the scope of this paper, stabilizing controllers are assumed to be operating.