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An adaptive order/state estimator for linear systems with non-integer time-varying order☆

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ABSTRACT

This paper proposes the design of a simultaneous order estimator and state observer for non-integer time-varying order linear systems. Several lemmas and theorems pertaining to the stability of variable order systems are provided first. Next, a theorem proposes an order/state estimator for linear variable order systems. Then, a simulation study is presented to verify the theoretical results.

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1. Introduction

Differential equations involving derivatives of quantities are the most common tools for describing dynamic events. A generalized version of calculus involves non-integer order for differentiation or integration, namely the fractional order or non-integer order calculus (Butzer & Westphal, 2000). Fractional order calculus, mostly developed in the 19th century, was considered a theoretical topic until recently when the increased flexibility, generality, and degrees of freedom in non-integer order differential equations proved them powerful in better describing certain real-world events as compared to integer-order differential equations. Interesting applications of such dynamics with real or even complex order have been reported in modeling, physics, and engineering (Podlubny, 1998; Sabatier, Agrawal, & Machado, 2007). The non-integer order dynamics have been used to model the memory in electronic devices (Maundy, Elwakil, & Gift, 2012), viscoelastic damping (Wharmby & Bagley, 2013), the human's ability to forget and remember (Tabatabaei, Yazdanpanah, & Tavazoei, 2013), and properties of tissue (Tabatabaei, Talebi, & Tavakoli, 2017a).

1.1. Prior art in variable order dynamics

The order of a non-integer order system is not limited to be a constant. It can vary depending on time, the states of the system, or even has its own dynamics. The concept of variable-order calculus was first developed in Samko and Ross (1993). Afterwards, the topic was studied from different aspects. In Lorenzo and Hartley (2002), the variable and distributed order operators and their properties are studied. Variable order derivative and its numerical approximation are investigated in Valério and Da Costa (2011). The variable order derivation operator is redefined in Sierociuk, Malesza, and Macias (2013a) for better accuracy in cases where the variable order is not a continuous function of time. This operator is used in Sierociuk, Malesza, and Macias (2013b) to form a switching order derivative operator. The problem of existence and uniqueness of the response of variable order differential equations has been studied in Razminia, Dizaji, and Majd (2012) and Zhang (2013). Extensions for the concepts previously introduced for traditional dynamics, such as variational calculus (Odzijewicz, Malinowska, & Torres, 2013a; Tabatabaei, Yazdanpanah, & Tavazoei, 2017b) and the Noether's theorem (Odzijewicz, Malinowska, & Torres, 2013b) have been also proposed for variable order dynamics. From the application sight of view, some evidences have been reported to relate the order of derivation to some physical quantities (Sheng, Sun, Coopmans, Chen, & Bohannan, 2011; Sierociuk, Podlubny, & Petras, 2013c). The main interpretation of the order is the memory. Such memory can belong to a physical system. in Sun, Chen, Wei, and Chen (2011), a study has been done to

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compare the effect of constant order versus variable order on the ability of a system to be influenced from the past, which is in fact, relating the memory of a system to its order. Order is used to measure the memory of a system in Du, Wang, and Hu (2013). The word memory can also be related to the human's mental ability to remember. It is shown in Tabatabaei, Yazdanpanah, Tavazoei, and Karimian (2012) that the order of an emotional dynamical system and the human's memory is strongly related. In Tabatabaei, Yazdanpanah, Jafari, and Sprott (2014), it is proven that the variable order of derivation is consistent with variable-length memory. Variable-order dynamical systems are used in Tabatabaei et al. (2014) to explain emotional behaviors of human caused by the effect of memory on context. There are some other applications and interpretations for the non-integer order. In Xu, Yang, Zhao, and Zhao (2015), the non-integer order derivation operator is used to improve the image quality by means of using spatial non-integer order derivative instead of integer order one for edge preserving and varying the order with respect to the location of each pixel. Describing state dependent viscoelasticity by the variable order (Tabatabaei et al., 2017a), describing two-dimensional cable equations (Bhrawy & Zaky, 2015), describing diffusion and subdiffusion in different cases (Chen, Zhang, & Zhang, 2013; Sun, Chen, & Chen, 2009), and explaining the behavior of ladders and nested ladders (Sierociuk et al., 2013c) are some other application of variable order dynamic systems.

1.2. Summary of contributions

Since the order is a key characteristic of a non-integer order system, for having a precise model describing specific dynamics, it is vital to estimate it with an acceptable level of precision. In most previous works involving variable order dynamics, the order is assumed to be known (Bhrawy & Zaky, 2015; Chen et al., 2013). Some researchers have tried to propose an estimation scheme for the order of a non-integer order system, assuming a **constant order**. In Hussain and Elbergali (1999), the order is treated just like other system constant parameters in the estimator which is not a real time approach. A numerical method is used in Victor, Malti, Garnier, and Oustaloup (2013) to find the constant order of a non-integer system. In Sierociuk and Dzielinski (2006), a discrete technique is used to estimate the constant order of a system using Kalman filter. In Rapaic and Pisano (2014), an adaptive estimation process is introduced to estimate the constant order of non-integer commensurate order systems. The approach proposed in the current paper differs from the above in the following aspects:

1. While the order is supposed to be constant in Rapaic and Pisano (2014), the order is allowed to be varying with time in this paper. This is a significant useful generalization. In fact, even when the order is a function of the states, it can be considered as an implicit function of time. Hence, the method proposed in this paper can be utilized in all cases involving varying order dynamics.

2. A definite convergence proof is established in this paper, guaranteeing that the estimation error for both order and states can be made arbitrarily small.

3. While the prior art only deals with the order and other system parameters, here, a state observer is designed for the cases where the states of the systems are not available and only the output can be measured.

4. Theorems 1–3, which act as intermediate results to prove the main result in Theorem 4, are useful for stability analysis of variable order systems and introduced for the first time in this paper.

5. Based on item 4 above, the order/state estimator proposed in Theorem 4 for variable-order systems includes a Luenberger-type state observer for regular integer-order systems. This means that the wealth of existing methods on observer gain selection, noise attenuation, etc. can be easily extended to the state observer part of the order/state estimator of a variable-order system.

6. The method presented in Rapaic and Pisano (2014) is designed for the single input single output systems, described in frequency domain. However, the novel method proposed here can be used in multi-input multi-output cases, as well.

The main achievement of this paper is a powerful estimator for the variable order of any linear system in a compact temporal interval $[0\ T]$, even when its states are not available. The estimator can be used to determine if a system is of constant or variable order or even, whether it is of integer or non-integer order. The estimation process is done in a real-time manner, so it can be used in adaptive control or model predictive control based approaches. Moreover, for the design of the proposed estimator, some new lemmas and theorems are proposed that are very useful in determining some of the most important properties of variable order systems including their stability. It is noteworthy to mention that the algorithm works for any given $T < \infty$. Hence, depending on the studied problem, when the final time is free, T can be set large enough to provide the required time for the convergence.

The rest of the paper is organized as follows: In Section 2, several definitions related to the non-integer order field and the problem statement will be presented. Section 3 is dedicated to proposing the adaptation rules for estimating the order and the states. After presenting some new and useful lemmas and theorems, the state observer will be designed and combined with the order estimator to build a comprehensive order/state estimator. In this Section, a case study is considered through a simulation study to show the effectiveness of the suggested methods. Finally, Section 4 concludes the paper.

2. Preliminaries

Definition 1. The definition of modified initialized left non-integer variable order derivation with respect to time in the sense of Caputo is

$$ {}_0^C D_t^{\alpha(t)} x(t) = \frac{1}{\Gamma(1-\alpha(t))} \int_0^t (t-\tau)^{-\alpha(t)} \frac{d}{d\tau} x(\tau) d\tau + \Psi_c^x(t) \qquad (1) $$
$$ 0 < \alpha(t) < 1, \forall t \geq 0 $$

where $\Gamma(.)$ is the extension of the factorial function to the non-integer arguments:

$$ \Gamma(z) = \int_0^\infty r^{z-1} e^{-r} dr \qquad (2) $$

$\Psi_c^x(t) = \frac{1}{\Gamma(1-\alpha(t))} \int_{-c}^0 (t-\tau)^{-\alpha(t)} \frac{d}{d\tau} x(\tau) d\tau$ is a decaying function capturing the effect of the values of the signal x before $t = 0$, supposing that x begins from $-c < 0$ (Lorenzo, Hartley, & Adams, 2013; Sabatier, Merveillaut, Malti, & Oustaloup, 2010). The use of this time varying initializing function is necessary to avoid discontinuity.

Definition 2. The inverse of the variable derivation operator is the variable order integration operator of the same order, could be interpreted as

$$ {}_0 I_t^{\alpha(t)} x(t) = \frac{1}{\Gamma(\alpha(t))} \int_0^t (t-\tau)^{\alpha(t)-1} x(\tau) d\tau \qquad (3) $$
$$ 0 < \alpha(t) < 1, \forall t \geq 0. $$

In fact;

$$ {}_0^C D_t^{\alpha(t)} I_t^{\alpha(t)} x(t) = x(t). \qquad (4) $$

Definition 3. Generally speaking, a non-integer order dynamic system is a set of coupled non-integer order differential equations

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