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Brief paper

Pareto suboptimal controllers in multi-objective disturbance attenuation problems ☆

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ABSTRACT

A multi-objective disturbance attenuation problem is considered as a novel framework for control and filtering problems under multiple exogenous disturbances. There are N potentially possible disturbance inputs of a system on each of which may act a disturbance from a certain class. A disturbance attenuation level is defined for each channel as an induced norm of the operator mapping signals of the corresponding class to the objective output of the system. Necessary conditions of the Pareto optimality are derived. It is established that the optimal solutions with respect to a multi-objective cost parameterized by weights from an N-dimensional simplex are Pareto suboptimal solutions and their relative losses compared to the Pareto optimal ones do not exceed $1-\sqrt{N}/N$. These results are extended to the case when the disturbances acting on different inputs are combined into coalitions. The approach is applied to multiple classes of L_2-bounded and impulsive disturbances for which the H_∞/γ_0 optimal controllers as the Pareto suboptimal solutions are synthesized in terms of linear matrix inequalities (LMIs). Illustrative examples demonstrate the effectiveness of the approach proposed.

1. Introduction

The multi-objective tradeoff approach is very useful in engineering practice when there are some competing objectives. It is well known that multi-objective problems traditionally are extremely difficult to solve and find the Pareto optimal set which is defined as a set of unimprovable alternatives. The paper of Mäkilä (1989) was one of the first in which the multi-objective problem for LQG objectives has been considered and the observer-based Pareto optimal solution has been obtained. Khargonekar and Rotea (1991b) have studied the multi-objective H_2 problem using Youla parametrization. They have presented the Pareto optimal controller in the terms of an infinite dimensional parameter Q. In special cases of a single input and multiple outputs or multiple inputs and a single output, the Riccati-based Pareto optimal solutions were derived. Hindi, Hassibi, and Boyd (1998) have extended the above approach based on Youla parametrization to multi-objective problems with H_2 and H_∞ criteria and a finite dimensional approximation for the Pareto optimal controllers has been proposed. For synthesizing these controllers Hindi et al. (1998) have used LMI characterizations for H_2 and H_∞ norms and ideas of Scherer, Gahinet, and Chilali (1997) for solving the mixed H_2/H_∞ problem.

The mixed H_2/H_∞ problem includes two different LMI constraints that make the problem non-convex. In order to recover convexity a common Lyapunov function was used by equating the Lyapunov matrices for the H_2 and H_∞ Bernstein and Haddad (1989), Chen and Zhou (2001), Doyle, Zhou, Glover, and Bodenheimer (1994), Khargonekar and Rotea (1991a) and Zhou, Glover, Bodenheimer, and Doyle (1994)). Of course, this leads to a conservative result. This conservatism was demonstrated by Molina-Cristobal, Griffin, Fleming, and Owens (2006) and Takahashi, Palhares, Dutra, and Goncalves (2004) with using genetic algorithms. Less conservative extensions of such results have been proposed by Ebihara, Peaucelle, and Arzelier (2015). Despite a huge amount of publications in the field of multi-objective control problem with H_2 and H_∞ criteria there are no results concerning the exact Pareto optimal set and a comparison of various approximate sets with the exact one.

This paper deals with multi-objective disturbance attenuation problems for disturbances from different classes. We consider linear systems with multiple disturbance inputs and a single objective output. A disturbance attenuation level which is the worst-case

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ratio of the objective output norm and a disturbance norm from the corresponding class is defined for each channel. The problem to be solved is to find a feedback controller minimizing these disturbance attenuation levels in sense of Pareto optimality.

The main contribution of this paper is a formulation and a proof of a necessary condition for Pareto optimality, synthesizing approximate Pareto optimal solutions and estimating a difference between exact and approximate solutions. Specifically, we introduce two performance measures, the optimal and suboptimal multi-objective costs. The minimization of the first one allows to cover the Pareto optimal solutions but is difficult to calculate, while the minimization of the second one can be performed by standard LMI procedures and allows to approximate the Pareto optimal set. The relative losses of the Pareto suboptimal solutions with respect to the Pareto optimal ones based on a comparison of the above costs turn out to be less than or equal to $1-\sqrt{N}/N$, where N is a number of criteria. This concept is extended to the case when the disturbances acting on different inputs are combined into coalitions; see also Balandin and Kogan (2016b).

In this paper, this approach is applied to multiple classes of L_2-bounded and impulsive disturbances with H_∞ and γ_0 norms as the disturbance attenuation levels. These classes are very important in the problems of optimal protection of the objects from shock and vibration; see, e.g., Balandin, Bolotnik, & Pilkey, (2001). The γ_0 norm was utilized by Balandin and Kogan (2008) and Iwasaki (1996) to characterize the worst-case ratio of the objective output norm and Euclidian norm of an intensity of impulsive disturbance. For these criteria the suboptimal multi-objective cost turns out to be the H_∞/γ_0 norm that is very closed to the mixed H_2/H_∞ norm.

The paper is organized as follows. Section 2 contains the formulation and the proof of the necessary condition for Pareto optimality in the general multi-objective disturbance attenuation problem including the case of possible coalitions of disturbances as well as estimating the relative losses of the Pareto suboptimal solutions. Section 3 is devoted to the multi-objective problems with H_∞ and/or γ_0 criteria. The H_∞/γ_0 norm and the corresponding worst-case disturbance are characterized in terms of LMIs. The Pareto suboptimal H_∞/γ_0 controllers are synthesized in Section 4. Section 5 provides with some illustrative examples and Section 6 concludes the paper.

2. Multi-objective disturbance attenuation problem

2.1. Pareto optimal and suboptimal solutions

Consider a linear time-invariant internally stable dynamic system parameterized by the parameter matrix Θ, in which $\xi_k \in R^{n_k}$, $k=1,\dots,N$ are the exogenous disturbance inputs, and $z \in R^{n_z}$ is the objective output. Associated to each channel from the input ξ_k to the output z is defined the disturbance attenuation level

$$J_k(\Theta) = \sup_{\|\xi_k\|_{\Xi_k}\neq 0} \frac{\|z\|}{\|\xi_k\|_{\Xi_k}}, \quad k=1,\dots,N, \tag{1}$$

where the disturbance $\xi_k=\xi_k(t)$ belongs to Ξ_k, one of the given classes of signals, and $z=z_k$ is the objective output in response to the disturbances $\xi_k(t)$ and $\xi_s(t)\equiv 0$ for all $s\neq k$ under zero initial conditions, $\|z\|$ is the norm in the z-space, and $\|\cdot\|_{\Xi_k}$ is the norm in Ξ_k. A key concept in multiobjective optimization is the Pareto optimal set. The set $\mathcal{P}=\{\Theta_P\}$ is the Pareto optimal if there is not a matrix Θ such that the inequalities $J_k(\Theta)\le J_k(\Theta_P)$, $k=1,\dots,N$, with at least one of the inequalities being strict, be valid. The problem to be considered is to characterize Pareto optimal solutions Θ_P, which is usually denoted as follows

$$\Theta_P = \arg\min_{\Theta}\{J_k(\Theta),\ k=1,\dots,N\}.$$

Such a problem can be regarded as a game between the nature which selects a signal from one of the specified classes to direct it at the corresponding channel and a person who chooses the matrix Θ in such a way to minimize his losses.

To this goal, let us define on the trajectories of the system under disturbances acting on all inputs simultaneously the multi-objective cost

$$J_\alpha(\Theta) = \sup_{\xi_k\in\Xi_k,\forall k} \frac{\|z\|}{\sum_{k=1}^N \alpha_k\|\xi_k\|_{\Xi_k}}, \tag{2}$$

where $\alpha=(\alpha_1,\dots,\alpha_N)$, $\alpha_k>0$, $\sum_{k=1}^N\alpha_k=1$. The necessary conditions for Pareto optimality is derived in the following theorem.

Theorem 1. *Let $(\gamma_1,\dots,\gamma_N)$ be a Pareto optimal point in the space of criteria and Θ_α minimize the multi-objective cost $J_\alpha(\Theta)$ under $\alpha_i=\gamma_i/\sum_{k=1}^N\gamma_k$. Then $\Theta_\alpha\in\mathcal{P}$ and $J_k(\Theta_\alpha)=\gamma_k$, $k=1,\dots,N$.*

Proof. Suppose that $\Theta_P\in\mathcal{P}$ and $J_k(\Theta_P)=\gamma_k>0$, $k=1,\dots,N$. From (1) it immediately follows that $\forall\,\xi_k\in\Xi_k$

$$\|z\| = \left\|\sum_{k=1}^N z_k\right\| \le \sum_{k=1}^N \gamma_k\|\xi_k\|_{\Xi_k} = \gamma\sum_{k=1}^N\alpha_k\|\xi_k\|_{\Xi_k},$$

where $\alpha_k=\gamma_k/\gamma$, $\gamma=\sum_{k=1}^N\gamma_k$. Here z_k is the objective output in response to the disturbance ξ_k from the class Ξ_k under $\xi_s=0$ for all $s\neq k$. Consequently, $J_\alpha(\Theta_P)\le\gamma$. Now, if Θ_α minimizes $J_\alpha(\Theta)$, we have $J_\alpha(\Theta_\alpha)=\gamma_\alpha\le\gamma$. In turn, this means that

$$\|z_k\| \le \gamma_\alpha\alpha_k\|\xi_k\|_{\Xi_k} = \gamma_\alpha\frac{\gamma_k}{\gamma}\|\xi_k\|_{\Xi_k} \le \gamma_k\|\xi_k\|_{\Xi_k} \quad \forall\,\xi_k\in\Xi_k$$

or, in other words, $J_k(\Theta_\alpha)\le J_k(\Theta_P)$, $k=1,\dots,N$. By the definition of the Pareto set, it can be satisfied only if $\Theta_\alpha\in\mathcal{P}$.

According to this statement, Pareto optimal solutions should be sought among the optimal solutions for the performance measure $J_\alpha(\Theta)$. However, due to the fact that finding the optimal solutions for this cost is a rather complex issue, we look at another family of one-criterion problems whose solutions are easy to find and allow us to estimate the boundary of the Pareto set in the space of criteria.

Namely, we introduce an auxiliary multi-objective cost

$$\Gamma_\alpha(\Theta) = \sup_{\xi_k\in\Xi_k,\forall k}\frac{\|z\|}{(\sum_{k=1}^N\alpha_k^2\|\xi_k\|_{\Xi_k}^2)^{1/2}} \tag{3}$$

which is the worst-case disturbance attenuation level for disturbances acting on all inputs. In the sequel, we will show the optimal solutions for this criterion can be effectively tackled for some classes of disturbances. From (3) it immediately follows that

$$\Gamma_\alpha(\Theta) \ge \max\{J_k(\Theta)/\alpha_k,\ k=1,\dots,N\}. \tag{4}$$

Since

$$\frac{\|z\|}{\sum_{k=1}^N\alpha_k\|\xi_k\|_{\Xi_k}} \ge \frac{1}{\sqrt{N}}\frac{\|z\|}{(\sum_{k=1}^N\alpha_k^2\|\xi_k\|_{\Xi_k}^2)^{1/2}},$$

we have $J_\alpha(\Theta)\ge(1/\sqrt{N})\Gamma_\alpha(\Theta)$. Let $J_k(\Theta_P)=\gamma_k>0$, $k=1,\dots,N$ for some $\Theta_P\in\mathcal{P}$. From Theorem 1 it follows that $J_\alpha(\Theta_\alpha)=\gamma_\alpha\le\gamma$, where Θ_α minimizes $J_\alpha(\Theta)$ for $\alpha_i=\gamma_i/\gamma$ and $\gamma=\sum_{k=1}^N\gamma_k$. Consequently, the following inequalities hold:

$$\gamma \ge J_\alpha(\Theta_\alpha) \ge (1/\sqrt{N})\Gamma_\alpha(\Theta_\alpha) \ge (1/\sqrt{N})\Gamma_\alpha(\Theta_\alpha^*)$$

where Θ_α^* minimizes $\Gamma_\alpha(\Theta)$. Now, in view of (4), we obtain for $k=1,\dots,N$

$$(1/\sqrt{N})J_k(\Theta_\alpha^*) \le (1/\sqrt{N})\Gamma_\alpha(\Theta_\alpha^*)\alpha_k \le \gamma\alpha_k = \gamma_k. \tag{5}$$

Since $\Theta_\alpha\in\mathcal{P}$, either there exists a set of indices s such that $J_s(\Theta_\alpha^*)>J_s(\Theta_\alpha)=\gamma_s$ whereas $J_k(\Theta_\alpha^*)\le J_k(\Theta_\alpha)=\gamma_k$ for all other