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Brief paper

Asynchronous periodic event-triggered consensus for multi-agent systems☆

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ABSTRACT

Towards a digital and flexible implementation of triggering conditions for multi-agent systems, we propose a sampled-data framework with discrete-time controllers and event detectors. An asynchronous sampling mechanism between agents is considered, where the sampling period and triggering parameters are allowed to be chosen by each agent independently. It is shown that consensus can be reached under the proposed control and communication schemes for graphs that contain a directed spanning tree. Simulation results are given to illustrate the utility of the proposed method.

1. Introduction

Distributed problem solving relies extensively on agents being able to communicate to neighbors about shared data (Meng, He, Teo, Su, & Xie, 2015). The communication cost should play a crucial role in determining the number of messages sent to/received by an individual agent. Fundamental continuous-time consensus algorithms are based upon the assumption of sufficiently large communication bandwidth (Li, Duan, & Lewis, 2014; Lin, Wang, Han, & Fu, 2014; Meng, Ren, & You, 2010; Olfati-Saber & Murray, 2004; Ren & Beard, 2008). On the other hand, discrete-time, or sampled-data consensus algorithms are based on periodic information exchange synchronously or asynchronously among the whole network (Meng, Meng, Chen, Dimarogonas, & Johansson, 2016; Xie, Liu, Wang, & Jia, 2009). Due to the success of event-triggered communication protocols in communication savings, periodic relaxation, and control update reduction for linear systems (Meng & Chen, 2012), they have been introduced to multi-agent systems for control update reduction (Dimarogonas, Frazzoli, & Johansson, 2012), and communication frequency reduction (Zhong & Cassandras, 2010), respectively. The literature on event-triggered consensus of multi-agent systems may be classified into two groups: state-dependent triggering condition (Chen, Hao, & Rahmani, 2014; Fan, Feng, Wang, & Song, 2013; Xie, Xu, Chu, & Zou, 2015), and time-dependent triggering condition (Seyboth, Dimarogonas, & Johansson, 2013; Wu, Meng, Xie, Lu, Su, & Wu, 2017). The state-dependent triggering condition was motivated by the counterpart in single-agent systems. By careful design of the state-dependent triggering condition, some papers are able to provide Zeno free results, that is, agents do not communicate an infinite number of times in any finite time period (Nowzari & Cortés, 2016). However, it is challenging to guarantee a positive minimum inter-event interval for the state-dependent triggering condition in multi-agent systems. This problem arises from the fact that an agent may reach consensus with its neighbors before the consensus of the entire network. In order to find a positive lower bound on inter-event times, the purpose of Kia, Cortés, and Martínez (2015) is to achieve bounded consensus rather than exact consensus. For exact consensus, this challenging problem of guaranteeing a positive lower bound on inter-event times is solved by the utilization of the sampled-data measurement technique, which has been used in single linear systems (Heemels, Donkers, & Teel, 2013). The fundamental works of synchronous periodic event-triggered control for multi-agent systems include the node-based sampled-data measurement (Meng & Chen, 2013) and edge-based sampled-data measurement (Xiao, Meng, & Chen, 2015), where the edge-based sampled-data measurement has also been extended to agents with double integrator dynamics (Cao, Xiao, & Wang, 2015). Moreover, the time-dependent triggering condition was also proposed to find a positive lower bound on inter-event times in multi-agent systems with the aid of time information

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(Seyboth et al., 2013). Note that most results in the literature focus on undirected and connected networks. There are also results on event-triggered consensus of multi-agent systems over directed graphs, such as switching graphs (Cheng, Kan, Klotz, Shea, & Dixon, 2017), weight-balanced, strongly connected digraphs (Nowzari & Cortés, 2016), and weight-balanced, recurrently jointly strongly connected digraphs (Kia et al., 2015).

In this paper, we provide a complete solution to the event-triggered consensus problem of multi-agent systems under the sampled-data framework. This framework enables the triggering condition to rule out Zeno behavior automatically and paves the way for digital implementation. Existing results on event-triggered communication mostly rely on either continuous sampling or synchronous periodic sampling. A substantial contribution beyond our previous efforts, and those of others is to study the consensus problem for multi-agent systems with asynchronous sampling. We achieve a completely distributed implementable result inspired by the clock synchronization technique (Kadowaki & Ishii, 2015). More specifically, each agent is able to determine its own sampling period and detection parameters without knowledge of any global information. The only information known to each agent is the number of neighbors. Consequently, a unified framework is established for the consensus problem of graphs containing a directed spanning tree. In summary, the proposed methods enable us to unravel several challenging problems in event-triggered consensus of multi-agent systems, such as Zeno free, asynchronous sampling, and reducible directed graphs.

2. Preliminaries

2.1. Notations

The set of $n \times n$ matrix is denoted by $\mathbb{R}^{n\times n}$. Every entry of the vector $\mathbf{1}$ is 1; and every entry of $\mathbf{0}$ is 0. The symbols $\lceil x \rceil$ and $\lfloor x \rfloor$ mean the ceiling function and floor function of the real number x, respectively. The relative complement of A in B is denoted by $B \setminus A = \{x \in B | x \notin A\}$.

2.2. Graph theory

Here we collect basic definitions about graphs and their algebraic properties. Further details can be found in Godsil and Royle (2001) and Diestel (2010).

A directed graph is a pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ which consists of a vertex set $\mathcal{V} = \{1, 2, \ldots, n\}$ and an edge set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ in which an ordered pair (i, j) means that agent j can receive information from agent i. If there exists an edge $(i, j) \in \mathcal{E}$, then we say that agent i is a neighbor of agent j. The set of neighbors of agent i is denoted by $\mathcal{N}_i$. A directed path from i to j in a directed graph is a sequence of edges starting with i and ending with j. A directed tree is a directed graph in which one vertex is designated as the root, which has a directed path to every other node. All other nodes have a unique parent. A directed spanning tree of a directed graph is a directed tree subgraph.

The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ of a directed graph is defined such that $a_{ij} = 1$ if $(v_j, v_i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. Define the Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{n\times n}$ as

$$l_{ii} = \sum_{j=1, j\neq i}^{n} a_{ij}, \qquad l_{ij} = -a_{ij}, i \neq j.$$

For a directed graph, $\mathcal{L}$ is not necessarily symmetrical.

2.3. Matrix theory

Here we provide some definitions and useful results used throughout the paper (Kadowaki & Ishii, 2015).

A *nonnegative* (resp. *positive*) *matrix* is a matrix in which all elements are equal to or great than zero (resp. greater than zero).

Definition 1. A nonnegative matrix $A \in \mathbb{R}^{n\times n}$ with the property that $A\mathbf{1} = \mathbf{1}$, that is, all its row sums are $+1$, is said to be a *row stochastic matrix*.

Definition 2. For a row stochastic matrix $A \in \mathbb{R}^{n\times n}$, the quantity

$$\begin{aligned}\mathscr{T}(A) &= \frac{1}{2} \max_{i,j} \sum_k \left|a_{ik} - a_{jk}\right| \\ &= 1 - \min_{i,j} \sum_k \min\left\{a_{ik}, a_{jk}\right\},\end{aligned}$$

is called *the coefficient of ergodicity* of A. If $\mathscr{T}(A) < 1$, the matrix A is called *scrambling*.

The operator $\mathscr{D}$ for a vector $x = [x_1 \cdots x_n]^T$ is defined as

$$\mathscr{D}(x) = \max_{i\in\mathcal{V}} \{x_i\} - \min_{i\in\mathcal{V}} \{x_i\}.$$

Lemma 3 (*Hartfiel, 1998*). *For any vectors v and w, $\mathscr{D}(v + w) \leq \mathscr{D}(v) + \mathscr{D}(w)$. For a row stochastic matrix A and a vector v, $\mathscr{D}(Av) \leq \mathscr{T}(A)\mathscr{D}(v)$. For any row stochastic matrices A and B, $\mathscr{T}(AB) \leq \mathscr{T}(A)\mathscr{T}(B)$.*

3. Problem formulation

We will give a unified framework for both undirected and directed networks.

Consider the single-integrator dynamics given by

$$\dot{x}_i(t) = u_i(t), \qquad i = 1, \ldots, n, \tag{1}$$

where $x_i \in \mathbb{R}$ is the scalar state, and $u_i \in \mathbb{R}$ is the control input of agent i. The goal of this paper is twofold: (1) to design a consensus algorithm to drive the agents in the network to their agreement point based on intermittent shared data; and (2) to design an event-driven protocol to mediate the communication between neighboring agents.

Continuous measurement is assumed in most papers for technical reasons. However, it has been pointed out that discrete measurement is more realistic (Åström & Bernhardsson, 2002), and it is indeed the case in real applications (Ploennigs, Vasyutynskyy, & Kabitzsch, 2009). Therefore, we assume that the triggering condition is evaluated periodically with a fixed period h_i for agent i, which produces two time sequences: sampling instants $\mathcal{S}_i = \{kh_i\}_{k\in\mathbb{Z}_{\geq 0}}$ and event instants $\mathcal{T}_i = \left\{t_k^i\right\}_{k\in\mathbb{Z}_{\geq 0}}$ with $t_0^i = 0$. The sampling instants exclusively refer to the time when agents access available local information, and perform event detection. Event instants correspond to the time instants when the communication actions occur, that is, agent i broadcasts its current sampled state to all agents who can receive information from agent i. For the agents who are not neighbors of any other agents, that is, the agents only receive information from their neighbors but do not need to broadcast any information, events instants are defined for the agents to update their own control law. The set of event instants is a subset of that of sampling instants, that is, $\mathcal{T}_i \subseteq \mathcal{S}_i$. With the aid of these two time sequences, we define

$$\hat{x}_i(t) = x_i\left(t_k^i\right), \qquad \text{for } t \in \left[t_k^i, t_{k+1}^i\right), \tag{2}$$

$$\bar{x}_i(t) = x_i(kh_i), \qquad \text{for } t \in [kh_i, kh_i + h_i). \tag{3}$$