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Technical communique

From structurally independent local LTI models to LPV model☆

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The local approach to linear parameter varying (LPV) system identification consists in interpolating individually estimated local linear time invariant (LTI) models corresponding to fixed values of the scheduling variable. It is shown in this paper that, without any global structural assumption of the considered LPV system, individually estimated local state-space LTI models do not contain sufficient information for determining similarity transformations making them coherent. It is possible to estimate these similarity transformations from input–output data under appropriate excitation conditions.

1. Introduction

Linear parameter varying (LPV) models provide an effective approach to handling nonlinear control systems (Lopes dos Santos, Perdicoulis, Novara, Ramos, & Rivera, 2012; Mohammadpour & Scherer, 2012; Sename, Gaspar, & Bokor, 2013; Tóth, 2010). Some successful methods for LPV system identification have been reported recently (Lopes dos Santos, Azevedo-Perdicoúlis, Ramos, Martins de Carvalho, Jank, & Milhinhos, 2011; Mercere, Palsson, & Poinot, 2011; Piga, Cox, Toth, & Laurain, 2015; Tóth, Laurain, Gilson, & Garnier, 2012; Van Wingerden & Verhaegen, 2009; Zhao, Huang, Su, & Chu, 2012). In the local approach to LPV system identification, interpolation is essential to establishing global models from a collection of locally estimated linear time invariant (LTI) models (De Caigny, Camino, & Swevers, 2011; De Caigny, Pintelon, Camino, & Swevers, 2014; Tóth, 2010). As LTI state-space models can be estimated in an arbitrary state basis, it is necessary to use a coherent collection of local models for the purpose of interpolation.

This paper is focused on the problem of making local state-space models coherent, *without treating the interpolation step*. Only *state-space* models are considered in this paper, as local model coherence is not relevant for other models. For shorter expressions, the words "state-space" will be omitted from terms like "local state-space model" and "LTI state-space model". In practice, interpolation is based on a finite set of local LTI models, each corresponding to a specific value of the scheduling variable $p(t)$, the main discussion of this paper is thus about the case where $p(t)$ evolves within a finite set, but its motivation is indeed with the perspective of interpolation for continuous values of $p(t)$.

It seems natural to transform all the local LTI models to some canonical form in order to make them coherent. *The main purpose* of this paper is to point out the fact that, in the local approach to LPV system identification, *structurally independent* local LTI models *themselves* do *not* contain sufficient information to determine similarity transformations making them coherent. However, locally estimated LTI models can be made coherent by making use of the information contained in some input–output data sequences across all the working points, notably with an algorithm initially introduced in the framework of piecewise linear hybrid systems (Verdult & Verhaegen, 2004; Zhang & Ljung, 2015).

Preliminary results of this work have been presented in Zhang and Ljung (2015), which are completed in the present paper with a rigorous proof of the main result.

2. Problem statement

Let $u(t) \in \mathbb{R}^q$ and $y(t) \in \mathbb{R}^s$ be respectively the input and the output at discrete time instant $t = 0, 1, 2, \ldots, p(t)$ be the scheduling variable evolving within a compact set $\mathbb{P}$. An LPV system is described by the state-space model

$$x(t+1) = A(p(t))x(t) + B(p(t))u(t) + w(t) \quad (1a)$$
$$y(t) = C(p(t))x(t) + D(p(t))u(t) + v(t) \quad (1b)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $A(p(t)), B(p(t)), C(p(t)), D(p(t))$ are matrices of appropriate sizes depending on $p(t) \in \mathbb{P}$, and $w(t) \in \mathbb{R}^n$, $v(t) \in \mathbb{R}^s$ are state and output noises with covariance matrices $Q(p(t)), R(p(t))$.

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Based on the fact that the LPV system (1) becomes an LTI system when the scheduling variable $p(t)$ is maintained at a fixed value, the following definitions aim at establishing a link between LPV and LTI models.

Consider a set of m LTI models indexed by the integer i:

$$x(t+1) = A_i x(t) + B_i u(t) + w(t) \tag{2a}$$
$$y(t) = C_i x(t) + D_i u(t) + v(t) \tag{2b}$$

characterized by matrices A_i, B_i, C_i, D_i of appropriate sizes, and noise covariance matrices Q_i and R_i.

The notation

$$\sigma_i \triangleq (A_i, B_i, C_i, D_i, Q_i, R_i) \tag{3}$$

will be used to denote the *matrices* characterizing the ith local LTI model (2), or the LTI model itself *by abuse of notation*. The set of LTI models will be denoted by

$$\Sigma = \{\sigma_i : i = 1, 2, \ldots, m\}. \tag{4}$$

Definition 1. A set of local LTI models

$$\Sigma^* = \{(A_i^*, B_i^*, C_i^*, D_i^*, Q_i^*, R_i^*) : i = 1, 2, \ldots, m\} \tag{5}$$

is called a *multi-snapshot* of the LPV system (1) for

$$p(t) \in \mathbb{P} = \{p_1, \ldots, p_m\} \subset \mathbb{\Pi}, \tag{6}$$

if

$$\begin{aligned} &A_i^* = A(p_i),\ B_i^* = B(p_i),\ C_i^* = C(p_i),\\ &D_i^* = D(p_i),\ Q_i^* = Q(p_i),\ R_i^* = R(p_i). \quad \square \end{aligned} \tag{7}$$

In the local approach to LPV system identification (De Caigny et al., 2011, 2014; Tóth, 2010), a set of locally estimated LTI models are interpolated to obtain a global model. As such local LTI models are typically estimated up to *different and arbitrary similarity transformations*, they do not constitute a multi-snapshot of the underlying LPV system in the sense of Definition 1. It is thus important to make the local models "coherent" before the interpolation step.

What does it mean by a "coherent" set of local LTI models with the perspective of their interpolation? The following auxiliary definition will be helpful.

Definition 2. The input–output behavior of a set of local LTI models $\Sigma = \{\sigma_1, \ldots, \sigma_m\}$ is the input–output behavior of the *multi-model switching system* consisting of the same set of LTI models, such that the ith LTI model σ_i is active when $p(t) = p_i \in \mathbb{P} = \{p_1, \ldots, p_m\}$, and at every transition between two LTI models, the initial state of the new active model is equal to the final state of the previous active LTI model. $\square$

For example, following this definition, the input–output behavior of a multi-snapshot Σ^* (see Definition 1) of an LPV system (1) is identical to the input–output behavior of the LPV system when $p(t)$ evolves within the restricted set $\mathbb{P}$.

If the interpolation of a set of local LTI models is expected to describe correctly an LPV system for all sequences of $p(t)$ within $\mathbb{\Pi}$, it should also be true in the particular case where $p(t)$ evolves within the restricted set $\mathbb{P} = \{p_1, \ldots, p_m\} \subset \mathbb{\Pi}$, including when $p(t)$ switches between different values within $\mathbb{P}$. It means that a "coherent" set of local LTI models should have the same input–output behavior as the underlying LPV system when $p(t) \in \mathbb{P}$, in the sense of Definition 2. This requirement will be satisfied by Definition 4 through Property 1.

Definition 3. Two LTI models $\sigma \triangleq (A, B, C, D, Q, R)$ and $\tilde{\sigma} \triangleq (\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}, \tilde{Q}, \tilde{R})$ are related by a *similarity transformation* characterized by an invertible matrix $T \in \mathbb{R}^{n\times n}$ and denoted as

$$\sigma \xrightarrow{T} \tilde{\sigma}, \tag{8}$$

if the matrices characterizing the two LTI models satisfy

$$\tilde{A} = TAT^{-1},\quad \tilde{B} = TB,\quad \tilde{C} = CT^{-1}, \tag{9a}$$
$$\tilde{D} = D,\quad \tilde{Q} = TQT^T,\quad \tilde{R} = R. \quad \square \tag{9b}$$

Definition 4. A set of LTI models

$$\tilde{\Sigma} = \{\tilde{\sigma}_i : i = 1, 2, \ldots, m\}$$

is said *coherent* with another set

$$\Sigma = \{\sigma_i : i = 1, 2, \ldots, m\},$$

if there exists an invertible transformation matrix $T \in \mathbb{R}^{n\times n}$, *common to the local models*, such that

$$\sigma_i \xrightarrow{T} \tilde{\sigma}_i \text{ for all } i = 1, \ldots, m. \tag{10}$$

This relationship between Σ and $\tilde{\Sigma}$ is then denoted by

$$\Sigma \overset{T}{\Longrightarrow} \tilde{\Sigma}. \tag{11}$$

The local LTI models $\tilde{\sigma}_i$ are simply said *coherent* when the reference model set Σ is obvious. $\square$

The relevance of this definition is justified by the following property, as discussed before Definition 3.

Property 1. *If a set of local models $\tilde{\Sigma}$ is coherent with the multi-snapshot Σ^* of the LPV system (1), then $\tilde{\Sigma}$ has the same input–output behavior (in the sense of Definition 2) as that of the LPV system (1) with the scheduling variable $p(t)$ restricted to the finite set $\{p_1, \ldots, p_m\}$ and with appropriate initial states.* $\square$

The proof of this property is trivial: under the assumed conditions, the LPV system (1) and its multi-snapshot Σ^* have the same input–output behavior, and the states of Σ^* and $\tilde{\Sigma}$ are related by the transformation matrix T.

Property 1 is a *necessary* condition that a relevant definition of local model coherence should satisfy. It does not exclude other possible definitions, notably those based on p-dependent state transformation matrices $T(p)$. Such considerations, related to LPV system equivalent state-space representations as investigated in Kulcsar and Tóth (2011), would be out of the scope of this technical communiqué.

In practice, local LTI models are estimated from a finite data sample subject to random uncertainties, thus the definition of coherent local models is understood in an approximative sense. If the estimation of each local model is consistent, then Definition 4 can also be understood for the limiting models when the data size for each local model estimation tends to infinity.

If some global structural assumptions of the matrix functions $A(p)$, $B(p)$, etc. were assumed, then they could be used to make estimated local LTI models coherent. This paper is focused on *structurally independent local LTI models*, as defined below.

Definition 5. A set of local LTI models σ_i are *structurally dependent* if their parametrizations are such that fixing the matrices $A_i, B_i, C_i, D_i, Q_i, R_i$ reduces the degrees of freedom of the matrices $A_j, B_j, C_j, D_j, Q_j, R_j$, for $j \neq i$; otherwise they are *structurally independent*. $\square$

For instance, if a set of local LTI models is parametrized such that all the matrices A_i share an equal entry at the same position, say $A_i(1,1) = A_j(1,1)$ for all $i, j = 1, 2, \ldots, m$, then the local models are structurally *dependent*. A less trivial counterexample will be given in Section 3.2 with Eq. (27).