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Semi-analytical minimum time solutions with velocity constraints for trajectory following of vehicles[☆]

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ABSTRACT

We consider the problem of finding an optimal manoeuvre that moves a car-like vehicle between two configurations in minimum time. We propose a two phase algorithm in which a path that joins the two points is first found by solving a geometric optimisation problem, and then the optimal manoeuvre is identified considering the system dynamics and its constraints. We make the assumption that the path is composed of a sequence of clothoids. This choice is justified by theoretical arguments, practical examples and by the existence of very efficient geometric algorithms for the computation of a path of this kind. The focus of the paper is on the computation of the optimal manoeuvre, for which we show a semi-analytical solution that can be produced in a few milliseconds on an embedded platform for a path made of one hundred segments. Our method is considerably faster than approaches based on pure numerical solutions, it is capable to detect when the optimal solution exists and, in this case, compute the optimal control. Finally, the method explicitly considers nonlinear dynamics, aerodynamic drag effect and bounds on the longitudinal and on the lateral acceleration.

1. Introduction

Over the past twenty years, the polar star for a large number of research activities has been how to introduce artificial intelligence into automobiles in order to make them safer, environment friendly and autonomous. The main actors of this impressive and consistent effort have been top academic institutions, research funding agencies such as DARPA and the European Commission, automotive industries and, more recently, important players in the Information and Communication Technologies area such as Google and Apple. Some tangible results of this activity are already available in modern commercial cars in the form of technological packages for lane keeping, autonomous braking, pedestrian detection and active cruise control. The next stride will likely push a new generation of cars straight into the realm of autonomous driving. This is far from being a remote possibility: starting from 2015 T cars allow the user to switch to autopilot mode, although under her/his legal responsibility for possible accidents (Greenblatt, 2016). Successful examples of cars driving autonomously for thousands of miles have been documented in the scientific literature (Broggi et al., 2013; Guizzo, 2011), but some crucial research problems are still there in search of cost effective and robust solutions. One of these is planning trajectories for vehicles manoeuvring at high speed, pushed to their dynamic limit and in the presence of moving or fixed obstacles. Trajectory planning in such extreme driving scenarios requires robust computational methods to produce a number of alternative feasible manoeuvres to choose from in a small time.

Related work. When the vehicle speed is high and the driving conditions are extreme, the application of a purely geometric approach such as optimal geometric path planning (Fraichard & Scheuer, 2004) falls short of the expectations and the dynamic constraints have to be put in their proper place (Sanfelice, Yong, & Frazzoli, 2014). In this context path planning solutions based on optimal control hold the promise to produce high quality manoeuvres that account for the vehicle dynamics and for the related constraints and can realistically be tracked by a vehicle (Bertolazzi, Biral, Da Lio, Saroldi, & Tango, 2010; Biral, Lot, Rota, Fontana, & Huth, 2012; Lot & Dall Bianco, 2016; Perantoni & Limebeer, 2014; Tavernini, Massaro, Velenis, Katzourakis, & Lot, 2013). The price to pay is the high computational cost incurred and the absence

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of any guarantee on the generation of feasible solutions. Both problems make such algorithms hardly a viable choice for real-time (reactive) planning or for the production of a large number of potential trajectories in a small time.

This consideration has motivated several authors to seek different solutions. One possible method is based on the fast generation of feasible kinematic trajectories *via* direct search and on the subsequent refinement of the solution by incorporating dynamic properties (Dolgov, Thrun, Montemerlo, & Diebel, 2010; Kuwata et al., 2009; Urmson et al., 2008). Such approaches have the considerable advantage to always produce feasible solutions, albeit not generally optimal. An interesting and new research area uses differential flatness to transform the problem from the canonical state space to flat output variables, where the solution is independent from the differential model equations (Mellinger & Kumar, 2011; Richter, Bry, & Roy, 2016). Flat methods require to approximate the optimal function on a polynomial basis and solve the associated NLP to optimise the target function (e.g. minimum time in our case). It is known from Cai, Yang, and Zhu (2016) that bang–bang problems are not particularly suitable for this method because of the intrinsic discontinuous nature of the solution, unless continuity is enforced. However, the main drawback of those solutions is that it is not possible to avoid purely numeric computations to deal with constraints that are computationally slower than analytic or semi-analytic solutions. A possible alternative to produce near-optimal solutions in a small amount of time with a guaranteed convergence is by a hierarchical approach. A *master* optimisation problem generates several alternative paths, reasoning at a geometric level, while a *slave* optimisation generates the optimal manoeuvres for each of the considered paths accounting for the dynamics of the vehicle and for the constraints. Based on the results of the slave, the master selects the path for which the generated trajectory has the best performance. By decoupling geometric from dynamic planning, it is possible to consider a variety of different constraints for each of the two (e.g., presence of obstacles in the geometric part or acceleration constraints in the dynamic part) putting in place the most appropriate solutions. On the contrary a monolithic formulation encompassing both the geometric and the dynamic part can potentially give rise to serious complexity and scalability issues. The available options for an effective design of a master algorithm are quite a few and range from stochastic sampling algorithms such as RRT/RRT* (Karaman & Frazzoli, 2011; Karaman, Walter, Perez, Frazzoli, & Teller, 2011; Kuffner & LaValle, 2000) to particle swarming (Qin, Sun, Li, & Cen, 2004), from graph based approaches (Rizano, Fontanelli, Palopoli, Pallottino, & Salaris, 2013) to potential fields (Barraquand, Langlois, & Latombe, 1992). Whatever the choice of the master algorithm (a first example implementing this idea of master/slave problem decomposition using RRT* can be found in Frego, Bevilacqua et al., 2016), the large number of times the slave problem has to be solved makes its efficiency key to the viability of the whole idea.

In this work, we propose a solution for *a minimum time manoeuvre over a given sequence of clothoid curves* for a car-like vehicle subject to acceleration constraints. This is a close suboptimal solution for a more general trajectory planning problem: moving a car-like vehicle in minimum time between two configurations. The convenience of choosing clothoids as basic motion primitive is derived from a few observations. One reasonable and widely adopted assumption is that the driver actuates the steering wheel without discontinuities, making the curvature of the path a continuous function (see, for instance, Fraichard & Scheuer, 2004). By choosing the simplest continuous curvature function (i.e., a piecewise linear function) the resulting path is a sequence of clothoids. The case of zero curvature (i.e., straight lines) and the case of constant curvature (i.e., arc of circle) are special cases of a clothoid. Such curves are known to be the fundamental building block for minimum time manoeuvres for the Dubins car (Dubins, 1957), i.e., a unicycle that moves at constant speed. Moreover, for an actual car vehicle having the velocity modelled as a linear ODE, it can be shown that the limitations of the lateral velocity produce exactly a clothoid. Finally, producing a clothoid that joins two points optimising some geometric cost is a problem for which efficient solutions exist (Bertolazzi & Frego, 2015). Other methods to produce a geometric path include fast generators of Kappa (Anderson, Beard, & McLain, 2005) and Gamma (Hota & Ghose, 2013) trajectories, which are made of sequences of straight segments and circular arcs. The discontinuous curvature thus obtained is more suitable for holonomic vehicles than for car vehicles. It has to be noted that our algorithm is capable to find the optimal speed profile for those kinds of paths too, since segments and circular arcs are particular cases of clothoids.

Paper contributions. In the paper, we will offer additional arguments and numeric examples to support our choice of using clothoids. Then, we will propose a *semi-analytical* solution to find the optimal manoeuvre for a single clothoid based on a direct application of the Pontryagin Maximum Principle. An important feature of our work is the explicit consideration of quadratic drag term in the longitudinal dynamics of the vehicle. This term is mandatory for the proper description of high speed manoeuvres and is usually managed *via* numeric integration. In our setting, the consideration of the term does not disrupt the analytical form of the solution, which is made of segments where the acceleration has to be maximum or minimum and segments in which it is given by a simple analytic expression. The switching points between the different segments are found *via* the solution of simple polynomial equations. The application of semi-analytical solutions for optimal control problems in the context of motion planning has been championed by Hauser & Saccon (2006) and Velenis & Tsiotras (2008), who studied the minimum time control strategy that accounts for the vehicle dynamics, speed and control constraints over a given trajectory. The authors resort to numeric solutions for forward and backward integration. With respect to this work, our analytic condition considers complex nonlinear dynamics and constraints (e.g., aerodynamic drag) and does not rely on any type of forward and backward numeric integration. In a different context Da Lio et al. (2015) propose semi-analytical solutions to represent the motor primitives at the basis of the complex manoeuvres generated by a human driver. However, the model they use does not consider acceleration and speed constraints and the cost function that they optimise is related to weighted minimisation of jerk and time without any constraints on states and controls whereas we consider a constrained pure minimum time problem. Another important contribution of our paper is an algorithm to construct the optimal solution for a sequence of clothoid segments starting from one, and a theorem stating the optimality of the result.

The efficiency of our solution makes us confident that our work can be credibly used as building block for the slave algorithm in the hierarchical scheme outlined above. Although the main motivations of the paper are rooted in the automotive domain, we expect a full applicability of our results in a variety of different applications (e.g., guidance of AGVs in industrial applications). Furthermore, our solution allows us to consider the manoeuvrability envelope (ME) of the vehicle in terms of g–g diagram (Biral, Bertolazzi, & Da Lio, 2014; Milliken & Milliken, 1995). The ME characterises all feasible manoeuvres and it condenses information on maximum achievable performance and on state reachability as well as human driving preferences (Bosetti, Da Lio, & Saroldi, 2014). This paper subsumes the preliminary results reported in Frego, Bertolazzi, Biral, Fontanelli, and Palopoli (2016) and extends those findings in these respects: 1. A deep analysis and a numerical proof of the validity of the adopted car-like model is now reported; 2. The optimal control problem has been extended from a single to an arbitrary number of clothoids (i.e., a sequence). This extension is non